1. Some fun with cross sections.

(a) Consider elastic scattering of two hard spheres of radius $a$ as in the lecture notes, but this time in the center of mass frame. Show that

$$\frac{d\sigma}{d\Omega'} = a^2,$$

so the scattering is isotropic in the center of mass frame. The prime on $d\Omega'$ is meant to indicate the center of mass frame.

(b) In the lab frame, one of the particles is at rest before the collision. By considering the transformation of the center of mass angles ($\theta', \phi'$) to the lab angles ($\theta, \phi$), transform the above cross section to the lab frame and show that the same result is obtained as in lecture.

2. Diffusion Equations. We’ve discussed particle diffusion and found that the particle flux density is related to the concentration gradient according to the transport equation

$$\mathbf{J}_n = -D \nabla n.$$

In coming up with this equation, we assumed that the system was in a steady state, but that’s not necessary, it’s also valid if the concentration and flux density are functions of time.

(a) If particles are conserved then a change in particle concentration in a region must result from particles crossing the boundary of the region. Show that

$$\nabla \cdot \mathbf{J}_n + \frac{\partial n}{\partial t} = 0.$$

Hint: you might find the divergence theorem useful. This equation expresses the conservation of particles. It is called a continuity equation.

(b) If the continuity equation is combined with the transport equation, one can obtain a partial differential equation for $n$ (not containing $\mathbf{J}$). What equation do you obtain? This is called the diffusion equation.

Comment: if the concentration is a linear function of position, the equation you just derived shows that the concentration is independent of time. So this is a steady state solution. At a local maximum, your equation should show the concentration decreases with time. Does it? Similarly at a local minimum, the concentration increases with time.

Keep going, this problem continues on the next page!
Another comment: A very crude approximation to a spatial derivative of $f$ is $f/L$ where $L$ is a characteristic spatial dimension of the system. A very crude approximation to a time derivative is $f/T$ where $T$ is a characteristic time scale of the system.

(c) Suppose that in lecture, I open a bottle of ammonia in the front of the room. Estimate how long it will take you to notice an appreciable odor if you’re sitting in the back of the room. This is a “make crude but reasonable estimates” problem! You might get a relatively long time compared to normal experience. But normal experience may also involve convection (air currents).


6. K&K, chapter 14, problem 6. The main point here is to solve for the velocity as a function of radius!