Homework 10 - Solutions

4th January 2005

Problem 1

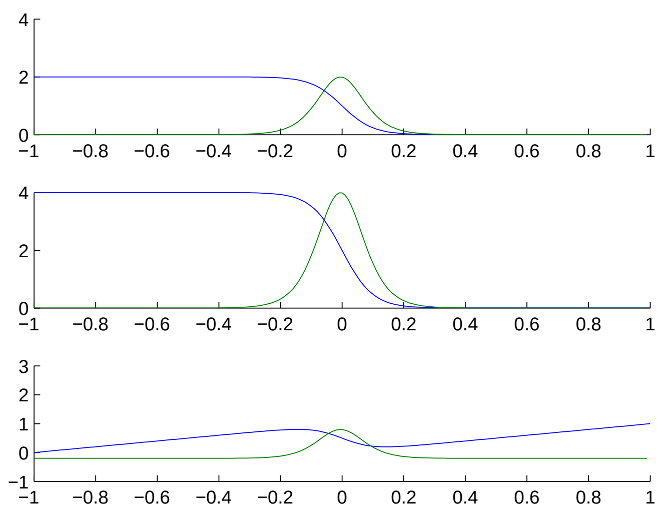

Figure 1: Voltage and electric field in p-n juction. The green curves (with peak) are the electric field. The n is on the left and p on the right.

To find the potential consider the following. The n-type of material has extra free electrons. That doesn't mean it's charged - it's neutral, but has extra free electrons. The p-type material has lack of eletrons - extra holes (it's also neutral). When we put them into contact the free electrons from n-type will diffuse into p-type and the p-type will be negatively charged.

This will happen until equilibrium is established - when chemical potential of diffusion is compensated by electric potential. The electric potential (for positive charge) decreases from left to right (because that's where it wants to go - down the hill). The electric field is $E = -\nabla V$. So we have the first figure. When we put positive potential on the left compared to the right, the electrons will be pulled out of the n-type to the left and more positive charge will be left on the left side of the barrier and similarly there will be more nagative charge on the right side of the barrier. Thus we have the second figure. Finally when we put negative potential on the left compared to the right, the electrons in n-type material will be pushed to the right, anihilate the holes accross the boundary and extra electrons will be supplied from the source to the left, and so the current will flow. When the current flows in the bulk of the material there has to be an electric field (e.g. V=RI, although here it's not linear). Thus we have the third figure.

Problem 2

(a) Let the impact parameter be b (distance between the lines on which centers move) and let suppose they scatter at the angle θ. We have

$$\frac{d\sigma}{d\Omega} = \frac{d\phi b db}{d\phi \sin\theta d\theta} = \frac{b db}{\sin\theta d\theta}$$

From the geometry (connect the centers of the spheres by line), $\sin(\theta/2) = b/2a$. This gives $b = 2a\sin(\theta/2)$, $db = a\cos(\theta/2)d\theta$ and so

$$\frac{d\sigma}{d\Omega} = \frac{2a\sin(\theta/2)a\cos(\theta/2)d\theta}{\sin\theta d\theta} = a^2$$

(b) We have

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega'}\frac{d\Omega'}{d\Omega} = \frac{d\sigma}{d\Omega'}\frac{d(\cos\theta')}{d(\cos\theta)}$$

Thus we need to relate the CM frame θ' to the lab frame θ. In the CM frame, when particle scatter it will get to some position $x' = l\cos\theta'$ and $y' = l\sin\theta'$ (after some time). Since the particle passed the distance l from the place of the collision, the coordinate x in the lab frame will be shifted by l. Thus the lab coordinates are $x = l + l\cos\theta'$ and $y = l\sin\theta'$. The new scattering angle is then

$$\tan\theta = \frac{y}{x} = \frac{l\sin\theta'}{l + l\cos\theta'} = \frac{2\sin(\theta'/2)\cos(\theta'/2)}{\cos^2(\theta'/2) - \sin^2(\theta'/2) + 1} = \tan(\theta'/2)$$

Thus $\theta' = 2\theta$. The crossection is

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega'}\frac{d\cos(2\theta)}{d\cos(\theta)} = a^2\frac{d(2\cos^2(\theta)-1)}{d\cos(\theta)} = 4a^2\cos(\theta)$$

Problem 3

(a) Number of particles in some region V is given by

$$N = \int_V d^3xn$$

Its change with time is given by the number of particles crossing the boundary, that is

$$\frac{\partial N}{\partial t} = -\int_{\partial V}\vec{J}\cdot d\vec{S}$$

Using divergence theorem we get

$$\int d^3x\frac{\partial n}{\partial t} = -\int_{\partial V}\vec{J}\cdot d\vec{S} = -\int d^3x\nabla\cdot\vec{J}$$

Since this is true for any volume, we can write

$$\frac{\partial n}{\partial t} = -\nabla\cdot\vec{J}$$

(b)

$$\frac{\partial n}{\partial t} = D\nabla^2 n$$

At local minimum, second derivative of concentration is positive and so $\frac{\partial n}{\partial t} > 0$.

(c) We can make a crude approximations that: $\frac{dn}{dt} \approx \frac{n}{T}$ and $\frac{d^2n}{dt^2} = \frac{n}{L^2}$. From diffusion equation we then get $T = L^2/D$. Taking $L = 5\text{m}$ and diffusivity from the web $D = 2\times10^{-5}\text{m}^2/\text{s}$ we get $T \approx 1.25\times10^6$. This is much higher then we expect from the experience, thus the transport occurs by some other mechanism too, such as convection (air currents).

Problem 4

This can be simply obtained from the following formulas in the book

$$K = C_vD = C_v\frac{1}{3}\bar{v}l$$

$$\sigma = \frac{nq^2\tau_c}{m}$$

$$C = \frac{3}{2}n$$

and approximating

$$\tau_c = l/\bar{v}$$

$$\bar{v}^2 \approx v_{rms}^2 = 3\tau/m$$

Putting all these together we get

$$\frac{K}{\tau\sigma} = \frac{3}{2q^2}$$

Problem 5

From chapter 7 we can calculate (or copy from table 7.1) that $T_F = 8.2 \times 10^4$K and $v_F = 1.56 \times 10^6$m/s. Because the room temperature is much smaller then this, the electron gas is degenerate, and we have to use the formulas for the degenerate electron gas.

(a) The heat capacity is

$$c_v = \frac{\pi^2}{2}\frac{N}{V}\frac{T}{T_F}k_B \approx 2 \times 10^4 \text{m}^{-3}\text{JK}^{-1}$$

(b) The electronic contribution to thermal conductivity is

$$K = Dc = c\frac{1}{3}v_f l \approx 4.2 \times 10^2 \text{J/mKs}$$

(c) The electrical conductivity is (we use $\tau_c \approx l/v_f$)

$$\sigma = 5.8 \times 10^7 \frac{1}{\text{ohm m}}$$

Problem 6 In the tube consider a cylinder of radius r centered at the center of the tube. The viscous force that acts on in equals $F = \eta(2\pi r L)dv/dr$. This cylinder keeps moving because there is the force of pressure that we are applying at the ends, which is $F = \pi r^2 p$. Equating these gives $dv/dr = pr/2L\eta$. Integrating this we get

$$v = \frac{p}{4L\eta}(a^2 - r^2)$$

Now we would like to find the amount of the liquid flowing. Consider the area between radii r and $r + dr$. The amount of liquid that flows through it in time dt is $dV = 2\pi r dr v dt$. Thus for the total flow of the liquid we get

$$\frac{dV}{dt} = \int_0^a 2\pi r v dr = \int_0^a \frac{p\pi}{2L\eta} r(a^2 - r^2) dr = \frac{p\pi a^4}{8L\eta}$$