PHYSICS DEPARTMENT, PRINCETON UNIVERSITY

PHYSICS 301 MIDTERM EXAMINATION

October 22, 2003, 10:00–10:50 am, Jadwin A06

This exam contains two problems. Work both problems. The problems count equally although one might be harder than the other. Do all the work you want graded in the separate exam books.

Write legibly. If I can't read it, it doesn't count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

1. Consider a system of N spin 1 particles with magnetic moments. These spins are in a magnetic field so that each spin has magnetic energy $-E$ when its magnetic moment is aligned with the field, energy $+E$ when anti-aligned and energy 0 when neither anti-aligned nor aligned (perpendicular). $E > 0$. There are weak interactions which keep the spins in equilibrium at temperature τ. Otherwise we can ignore interactions of the spins with each other and anything else but the magnetic field. (For example, take the particles to be at fixed positions so any thermal effects due to the center of mass motions of the particles can be ignored, etc.)

(a) Without elaborate calculation, determine the energy, U, and the entropy σ of this system at very low temperatures ($\tau \to 0$). Also, without elaborate calculation, determine the energy and entropy at very high temperatures ($\tau \to \infty$).

(b) What is the partition function of this system at temperature τ? Hint: there's an easy way and a hard way to do this.

(c) Find the free energy, entropy and energy of this system at temperature τ.

2. Consider a membrane (a two dimensional surface like a drumhead). We are interested in the low temperature thermal energy content and heat capacity of this membrane due to vibrational modes in which the displacement of the surface is perpendicular to the surface (also like a drum!). For convenience, imagine that the membrane is a square of side L with area $A = L \times L$ and it is fixed at the edges. The speed of waves on the membrane is v which we take to be independent of frequency.

(a) For a standing wave mode of frequency ω, what is the average energy when the mode is in equilibrium at temperature τ. Hint: each standing wave mode is a harmonic oscillator with discrete energy levels. We can call the quanta of these vibrations "drumons" (bad joke)!

(b) How many modes ($n(\omega)$) are there between frequencies ω and $\omega + d\omega$?

(c) What are the low temperature thermal energy content and heat capacity of the membrane? You may come up with an integral that's non-trivial. If so, put it in dimensionless form and call its value I.