PHYSICS DEPARTMENT, PRINCETON UNIVERSITY

PHYSICS 301 MIDTERM EXAMINATION

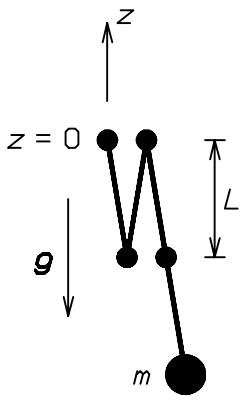
October 20, 2004, 10:00–10:50 am, Jadwin A06

This exam contains two problems. Work both problems. The problems count equally although one might be harder than the other. Do all the work you want graded in the separate exam books.

Write legibly. If I can't read it, it doesn't count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

1. Molecules in a polymer are long "chains" which can tangle up or stretch out. As a very simple model, consider a chain with a large number, N, of links of length L. For convenience, take N to be an even number. One end of the chain is fastened to a fixed support and a mass m is attached to the other end of the chain. The links are massless and the whole system is in a gravitational field $\boldsymbol{g}$ pointing down. In this particular chain, molecular forces allow the "hinges" between links to be completely folded or completely straight, so all the links are vertical as shown schematically in the figure. Take the gravitational energy to be zero when the mass is even with the support. Also, ignore all forms of energy other than gravitational energy and weak interaction energy with a heat bath at temperature τ.



(a) Without elaborate calculation, determine the average energy, U, entropy, σ, and free energy, F, at very low temperatures ($\tau \to 0$).

(b) Again without elaborate calculation, determine the energy, entropy, and free energy at very high temperatures.

(c) What is the partition function for this system at (any) temperature τ?

(d) What are the free energy, energy and entropy of this system at temperature τ?

2. Sometimes electrons are confined to a plane, but otherwise free to move in the plane. Consider a system containing N electrons of mass m confined to a plane square of side L and area $A = L \times L$. We completely ignore the motion perpendicular to the plane and the forces confining the electrons to the plane. Also ignore interactions between electrons. Assume the electrons are in thermal contact with a bath at temperature τ. Remember that electrons have spin 1/2.

(a) Determine the density of states. That is, determine the number of states per unit energy, where the energy, ϵ, is the kinetic energy of the two dimensional motion of the electron in the plane.

(b) Suppose the electrons are cold ($\tau \to 0$). What is the chemical potential (Fermi energy)?

(c) What is the total kinetic energy of this two dimensional gas of electrons?