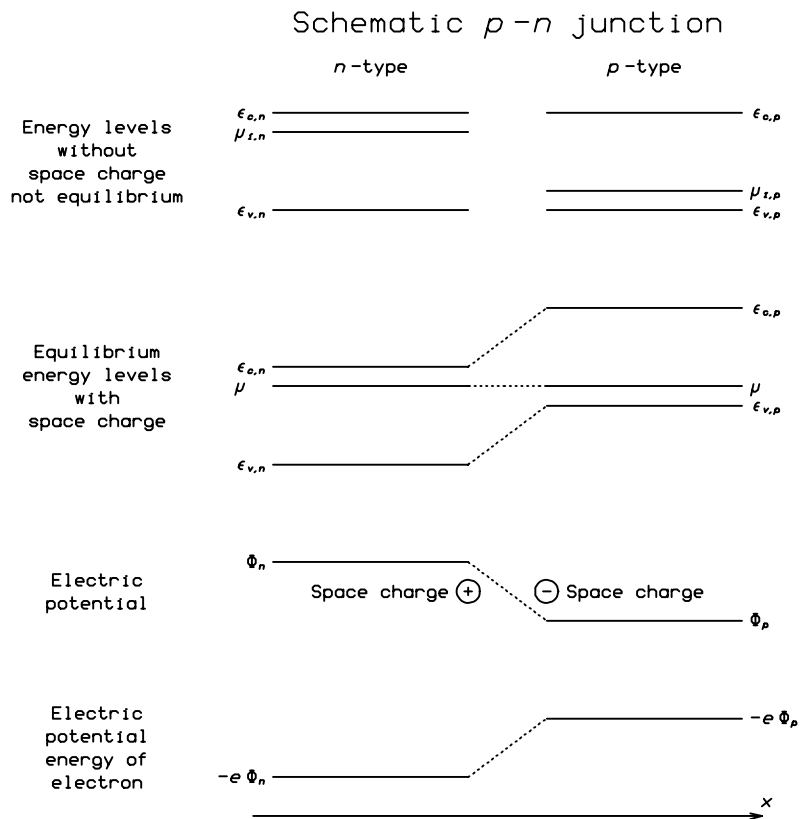


Reading

Finish K&K chapter 13, K&K chapter 14.

The Depletion Region in a  $p$ - $n$  Junction

The figure is a schematic of our view of a  $p$ - $n$  junction up to this point. The  $n$ -type material is on the left and the  $p$ -type is on the right.  $x$  is a spatial coordinate increasing from left to right. At the top are shown the energy levels one would have before the flow (diffusion) of electrons and holes to set up a space charge density which creates the electric potential step we are discussing. This is **not** an equilibrium configuration. Next is a diagram of the energy levels in the equilibrium configuration after the potential step has been established. The dotted lines indicate that the levels must connect from one side to the other, but to know how they connect, we will have to do some work! The chemical potential is now constant across the junction and all energy levels get a step as the junction is crossed. The next diagram shows the electric potential. The  $\oplus$  and  $\ominus$  symbols indicate the location of positive and negative space charge. Finally, the bottom diagram shows the electric potential energy of an electron. This is the potential curve, but inverted due to the negative charge of the electron.



As the electric potential curve is drawn, there is an infinitely thin plane of positive charge (holes) in the  $n$ -type semiconductor and a plane of electrons in the  $p$ -type semiconductor. (This gives a constant slope to the potential between the two planes.) Such a configuration is also not an equilibrium configuration. Instead, the positive and negative charge will spread out (that's why it's called space charge) in a region around the junction. To determine the space charge density, we must solve Gauss' law (from E&M) in conjunction with the thermodynamic constraints. We will work through a simple model to see how it goes. We will assume that nothing depends on  $y$  or  $z$ . Also, we assume the

doping changes abruptly from  $n$ -type to  $p$ -type at  $x = 0$ . In a real semiconductor, there is some diffusion of dopants, so the change from  $n$ -type to  $p$ -type can't be instantaneous. However, if it occurs in a distance shorter than the *depletion length* (coming soon), then our model of an abrupt change is a good approximation.

The electron and hole concentrations will be a function of  $x$  as will the energies of the band edges and the electric potential. Only the chemical potential is independent of  $x$ , once equilibrium has been established.

Recall from Gauss' law,

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0},$$

written for SI units, with  $\rho$  the charge density,  $\epsilon_0$  the permittivity, and  $\mathbf{E}$  the electric field. Note: if you took Physics 104, you learned the integral form of Gauss' law,

$$\oint_{\text{Closed Surface}} \mathbf{E} \cdot \mathbf{n} dA = \frac{1}{\epsilon_0} \int_{\text{Enclosed Volume}} \rho dV.$$

The integral and differential forms can be derived from one another using the divergence theorem (which you should have seen in a math class by now!)

$$\oint_{\text{Closed Surface}} \mathbf{X} \cdot \mathbf{n} dA = \int_{\text{Enclosed Volume}} \nabla \cdot \mathbf{X} dV,$$

where  $\mathbf{X}$  is any vector field. The electric potential is defined for static fields so that

$$\mathbf{E} = -\nabla\Phi,$$

which means

$$\nabla^2\Phi = -\frac{\rho}{\epsilon_0}.$$

If we specialize all this to our case, our fields are a function of  $x$  only, and we must use the permittivity of the semiconductor rather than that of free space. We have

$$\frac{d^2\Phi(x)}{dx^2} = -\frac{1}{\epsilon} \rho(x).$$

What is the charge density? In the  $n$ -type semiconductor, at large distances from the junction, there is a concentration  $n_d$  of positive charge and an equal concentration  $n_e$  of negative charge, so the semiconductor is neutral,  $\rho = 0$ . As we get close to the junction (but stay on the  $n$ -type side),  $n_d$  remains constant, but  $n_e$  decreases (fewer electrons is the same as more holes). One way to think of the decrease in the electron concentration is that it occurs because the conduction band edge,  $\epsilon_{c,n}$  is raised, relative to the chemical potential,  $\mu$ , by the addition of the electric potential energy of the electrons. That is, without an electric potential, we have

$$n_d = n_e = n_c e^{-(\epsilon_c - \mu)/\tau}.$$

with an electric potential, the difference between  $\epsilon_c$  and  $\mu$  widens and the electron concentration decreases,

$$n_e = n_c e^{-(\epsilon_{c,n} - e\Phi - \mu)/\tau} = n_d e^{e\Phi/\tau}.$$

The charge density is then

$$\rho(x) = e(n_d - n_e) = en_d \left(1 - e^{e\Phi/\tau}\right),$$

and Gauss' law becomes

$$\frac{d^2\Phi(x)}{dx^2} = -\frac{en_d}{\epsilon} \left(1 - e^{e\Phi/\tau}\right).$$

We've found the differential equation that  $\Phi$  must satisfy.

We can integrate this equation with the following trick. Multiply both sides by  $2\frac{d\Phi}{dx}$ ,

$$2\frac{d\Phi}{dx} \frac{d^2\Phi(x)}{dx^2} = -\frac{en_d}{\epsilon} 2\frac{d\Phi}{dx} \left(1 - e^{e\Phi/\tau}\right),$$

or

$$\frac{d}{dx} \left(\frac{d\Phi}{dx}\right)^2 = -\frac{2en_d}{\epsilon} \frac{d}{dx} \left(\Phi - \frac{\tau}{e} e^{e\Phi/\tau}\right).$$

At this point we need to think about the boundary conditions, that is, the values of  $\Phi$  at the limits of integration and in fact, we need to pick good limits of integration. To start with we assume that the semiconductor extends far enough away from the junction that the effects of the junction become negligible (that is,  $n_e \rightarrow n_d$ ). In this case, we might as well assume it extends to  $x = -\infty$  where we take the potential to be zero  $\Phi(-\infty) = 0$ . Also,  $x = -\infty$  will be one of our limits of integration. We will take the other limit to be  $x = 0$ . At this position,  $\Phi(0) = -\Delta V_n$ , where  $\Delta V_n$  is that part of the potential difference  $\Delta V$  that occurs in the  $n$ -type material. When we integrate, we will have to evaluate  $d\Phi/dx = -E_x$  at the limits of integration. We take  $E_x(-\infty) = 0$ , and represent by  $E$  the value of the electric field at  $x = 0$ . Then

$$(-E)^2 - 0 = -\frac{2en_d}{\epsilon} \left[ \left(\Phi(0) - \frac{\tau}{e} e^{\Phi(0)/\tau}\right) - \left(\Phi(-\infty) - \frac{\tau}{e} e^{\Phi(-\infty)/\tau}\right) \right],$$

or

$$E^2 = -\frac{2en_d}{\epsilon} \left(-\Delta V_n - \frac{\tau}{e} e^{-e\Delta V_n/\tau} + \frac{\tau}{e}\right).$$

Now,  $e\Delta V_n$  is of the same order as the energy gap which we're assuming is much bigger than  $\tau$ . This means the exponential above can be ignored and we have

$$E = \pm \sqrt{\frac{2en_d}{\epsilon} \left(\Delta V_n - \frac{\tau}{e}\right)},$$

and we must choose the positive sign since the electric field points in the positive  $x$ -direction at the junction. We've obtained a relation between the electric field at the junction and

part of the voltage drop across the junction. If we do the same arithmetic for the  $p$ -type material, the result is

$$E = \sqrt{\frac{2en_a}{\epsilon} \left( \Delta V_p - \frac{\tau}{e} \right)}.$$

It's the same electric field whether we calculate it from the  $n$ -type side or the  $p$ -type side, so we have

$$E^2 = \frac{2en_d}{\epsilon} \left( \Delta V_n - \frac{\tau}{e} \right),$$

$$E^2 = \frac{2en_a}{\epsilon} \left( \Delta V_p - \frac{\tau}{e} \right),$$

or

$$\frac{\epsilon E^2}{2e} \frac{1}{n_d} = \Delta V_n - \frac{\tau}{e},$$

$$\frac{\epsilon E^2}{2e} \frac{1}{n_a} = \Delta V_p - \frac{\tau}{e},$$

add

$$\frac{\epsilon E^2}{2e} \left( \frac{1}{n_d} + \frac{1}{n_a} \right) = \Delta V - \frac{2\tau}{e},$$

or

$$E = \sqrt{\frac{2e}{\epsilon} \frac{n_d n_a}{n_d + n_a} \left( \Delta V - \frac{2\tau}{e} \right)}.$$

We know the electric field at the junction in terms of the potential drop across the junction, the temperature, and the “givens.”

Returning to our differential equation for  $\Phi(x)$ , we were able to integrate it once, but so far as I know, it can't be integrated in closed form to get  $\Phi$ . Numeric integration is required to get  $\Phi(x)$  and from  $\Phi(x)$  one can calculate the electron concentration using

$$n_e(x) = n_d e^{e\Phi(x)/\tau}.$$

The electron concentration is essentially 0 at the junction and it rises to  $n_d$  as one moves away from the junction. We say that electrons are depleted at the junction. (And in the  $p$ -type material, holes are depleted at the junction.) To get a handle on how far from the junction the effects of the junction extend, we imagine that the electron density is 0 for some distance  $w_n$  in the  $n$ -type semiconductor. We ask what value of  $w_n$  is required in order that we have the same electric field at the junction as the electric field we calculated in the previous paragraph. We apply Gauss' law in integral form to this region by drawing a box with unit area perpendicular to  $x$  with one face at  $x = -w_n$ , where the electric field

is 0 and the other face at  $x = 0$  where the electric field is  $E$  calculated above. The charge in this box is  $n_d w_n e$ , and this divided by the permittivity must be  $E$ . So

$$w_n = \frac{\epsilon E}{e n_d} = \sqrt{\frac{2\epsilon}{e} \frac{n_a}{n_d(n_d + n_a)} \left( \Delta V - \frac{2\tau}{e} \right)}.$$

One expects that it will take several times  $w_n$  for the electron concentration to rise from 0 at the junction to  $n_d$  inside the  $n$ -type region. Similarly, the depletion length for the holes is

$$w_p = \frac{\epsilon E}{e n_a} = \sqrt{\frac{2\epsilon}{e} \frac{n_d}{n_a(n_d + n_a)} \left( \Delta V - \frac{2\tau}{e} \right)},$$

and the total depletion length is

$$w = w_n + w_p = \sqrt{\frac{2\epsilon}{e} \frac{n_d + n_a}{n_d n_a} \left( \Delta V - \frac{2\tau}{e} \right)} = \frac{2(\Delta V - 2\tau/e)}{E}.$$

To calculate a representative number, we take  $n_d = n_a = 10^{15} \text{ cm}^{-3}$ ,  $\epsilon = 10\epsilon_0$ , and  $\Delta V - 2\tau/e = 1 \text{ V}$ . We find,  $E = 1.34 \times 10^4 \text{ V cm}^{-1}$ . This is  $\sqrt{10}$  smaller than the number quoted in K&K. It appears that K&K may have left out the dielectric constant when computing the electric field (the 10 in  $\epsilon = 10\epsilon_0$ ). The characteristic width of the depletion region is  $w = 1.49 \times 10^{-4} \text{ cm}$ . Remember this is the width assuming that the depletion is 100% over this range and 0 outside the range. The depletion actually goes from 100% to 0 gradually over several times this distance.

### A Reverse Biased $p$ - $n$ Junction

Let's connect the  $p$ - $n$  junction to an external voltage source so that the positive terminal is connected to the  $n$ -type material and the negative terminal to the  $p$ -type material. The external voltage adds to the potential step at the junction. To see this consider the following. The positive terminal connected to the  $n$ -type material attracts electrons from the semiconductor. Since there are plenty of electrons in the  $n$ -type material, the electrons flow into the positive terminal and they keep an almost constant potential in the  $n$ -type material. But what happens at the junction? To replenish the electrons, we need electrons to flow across the junction from the  $p$ -type material. But the  $p$ -type material doesn't have any electrons! Of course the same statements work for the holes. Holes are attracted by the negative voltage, the plentiful holes in the  $p$ -type material keep the potential almost constant in the bulk of the  $p$ -type material and to replenish the holes, we need holes to flow from the  $n$ -type material where there aren't any. What happens is that the electric field at the junction gets bigger—we do all the same calculations but with the external voltage plus  $\Delta V$  in place of  $\Delta V$ —and the depletion region gets wider. The extra voltage drop is taken almost entirely at the junction and after the initial transient to establish equilibrium, there is almost no current flow.

### A Forward Biased $p$ - $n$ Junction

Let's connect the  $p$ - $n$  junction in the opposite sense. We connect the external negative voltage to the  $n$ -type semiconductor and the positive voltage to the  $p$ -type semiconductor. Now what happens? Electrons are repelled from the negative terminal, so an electron current travels from the negative terminal toward the junction. (As electrons leave the region of the negative terminal, more are supplied by the negative terminal.) The electrons forced into the depletion region can recombine with the holes producing a smaller depletion region. This means that the potential drop across the junction is smaller than in the equilibrium open circuit condition. This means that electrons will diffuse across the junction to re-establish the equilibrium potential drop. Electrons that diffuse across recombine with holes on the  $p$ -side. These holes have come from the positive terminal and either recombine with the electrons that have diffused to the  $p$ -side or they diffuse to the  $n$ -side and recombine with electrons that have come from the negative terminal. Whew! In any case, the potential drop across the junction and the width of the depletion region are both smaller than in the equilibrium case. Electrons and holes are diffusing across the junction and recombining and there is a current of electrons going from the negative terminal to the junction (which is an electric current going from the junction to the negative terminal) and a current of holes (and electric current) going from the positive terminal to the junction. This is not an equilibrium situation!

Since it's not an equilibrium situation, it's not really clear what to do about the chemical potential. The chemical potential must not be uniform since we have flowing electrons and holes. One approach is to say that the electrons in the conduction band

follow equilibrium Fermi-Dirac statistics as do the holes in the valence band, but the electrons and holes aren't in equilibrium with each other. Then one can use *quasi-Fermi levels*: separate chemical potentials for the valence and conduction bands (which also depend on position). Then the electron distribution in the conduction band is

$$f_c = \frac{1}{1 + e^{(\epsilon - \mu_c)/\tau}},$$

with a similar expression involving  $\mu_v$  for the valence band. This approach allows for an increase in the electron concentration in the  $n$ -type region (due to the electrons flowing in from the negative terminal) and an increase in the hole concentration as well, due to the holes diffusing across the junction and past the depletion region (not all will recombine in the depletion region). Also, if the electron concentration is increased, the hole concentration must increase if the material is to be electrically neutral.

Electrons diffuse from high chemical potential to low chemical potential. Since this is a non-equilibrium process, we don't know all that much about what happens. So we make the reasonable assumption that the electron (not electric) current, which is the number of electrons crossing a unit area in a unit time, is proportional to the negative of the gradient of the chemical potential as well as the electron concentration, so

$$\mathbf{j}_e \propto -n_e \nabla \mu_c,$$

where  $\mathbf{j}_e$  is the electron current. To get electric current we need to multiply by  $-e$ , so

$$\mathbf{J}_e \propto +en_e \nabla \mu_c.$$

Now we absorb  $e$  into the proportionality constant which is called the electron mobility  $\tilde{\mu}_e$ , which has nothing directly to do with the electron chemical potential. Then the electric current carried by electrons is

$$\mathbf{J}_e = \tilde{\mu}_e n_e \nabla \mu_c.$$

At this point the electron mobility is to be regarded as an experimentally determined parameter characteristic of the material and temperature, etc. With luck, we will learn more about mobility when we discuss chapter 14. If the electron concentration is classical, then

$$n_e = n_c e^{(\mu_c - \epsilon_c)/\tau},$$

so

$$\mu_c = \epsilon_c + \tau \log \frac{n_e}{n_c},$$

and the electric current carried by electrons in the conduction band is

$$\mathbf{J}_e = \tilde{\mu}_e n_e \nabla \epsilon_c + \tilde{\mu}_e \tau \nabla n_e.$$

The conduction band edge is equal to a constant plus  $-e\Phi$ , so  $\nabla\epsilon_c = -e\nabla\Phi = e\mathbf{E}$ , and

$$\mathbf{J}_e = e\tilde{\mu}_e n_e \mathbf{E} + eD_e \nabla n_e ,$$

where the diffusion coefficient is defined by  $D_e = \tilde{\mu}_e \tau / e$ . Again, we'll learn more about diffusion coefficients later. For now it can be regarded as an experimentally determined parameter.

The upshot of all this is that the electron current in the conduction band arises from an electric field (an electric potential gradient) and from a concentration gradient. Similarly, the hole current in the valence band is

$$\mathbf{J}_h = e\tilde{\mu}_h n_h \mathbf{E} - eD_h \nabla n_h .$$

A little thought will show that the signs are correct. Holes, having positive charge, travel in the direction of the electric field and contribute a current in that direction. They travel opposite to the hole concentration gradient and, having positive charge, contribute an electric current opposite to the hole concentration gradient.

Throughout most of the semiconductor, the field points from the  $p$ -type to the  $n$ -type region and there is very little concentration gradient, so it is the electric field term which is responsible for the electric current. At the junction, in the depletion region, the electric field term points the other way! The same direction as it pointed in the static, equilibrium case. (Unless we apply a whopping external field!) However, in the neighborhood of the junction, the concentration gradients are large and it is the diffusion due to these gradients that carries the current across the junction.