

1. This is similar to K&K, chapter 7, problem 2. The goal is to work out the Fermi energy and the ground state energy of a completely degenerate relativistic Fermi gas. The calculation should proceed just as the calculation for the non-relativistic Fermi gas in the lecture notes except that one takes the energy to be given by  $\epsilon = pc$  rather than  $\epsilon = p^2/2m$ .

Of course, the very low energy states may not actually be relativistic, but if the Fermi energy turns out to be many times the rest mass energy, then almost all the particles are relativistic and we don't make an appreciable error if we assume that all of them are relativistic.

You should be able to show that the Fermi and ground state energies are

$$\epsilon_F = \pi \hbar c \left( \frac{3n}{\pi} \right)^{1/3} \quad \text{and} \quad U_0 = \frac{3}{4} N \epsilon_F .$$

With these results modify our treatment of the electron and proton concentration in a neutron star. As always, make suitable approximations. What do you get for the relative fraction of electrons and protons?

2. K&K, chapter 7, problem 5. Note that  ${}^3\text{He}$  has a complicated phase diagram at low temperatures and can even become a superfluid. The 1996 Physics Nobel Prize was awarded to Lee, Osheroff, and Richardson for their discovery of superfluidity in  ${}^3\text{He}$ .

3. K&K, chapter 7, problem 6.

4. K&K, chapter 7, problem 8. Note that Figure 7.19 shows the calculated heat capacity curve above  $\tau_e$  as well as below  $\tau_e$  where you are asked to calculate it. How would you calculate the energy, heat capacity, and entropy above  $\tau_E$ ? (Note that you're not being asked to do the calculation, just outline the calculation!)

5. K&K, chapter 7, problem 12.

6. In the *Physics Today* article by Collins, August, 1995, vol. 48, no. 8, p. 17, it is stated that a Bose-Einstein condensate of about 2000  ${}^{87}\text{Rb}$  atoms forms at a temperature of  $170 \times 10^{-9}$  K. Use our theory to estimate the concentration of Rubidium atoms in the sample. What is the density? Assuming a spherical sample, what is the radius?

Note that K&K, chapter 7, problem 10 hints at an upper limit for the mass of white dwarf stars. Beyond this limit, about  $1.4M_\odot$ , gravity overwhelms the degeneracy pressure of the electrons and the white dwarf collapses (probably to explode as a supernova of "type Ia"). Similarly, there is a (more uncertain) upper mass limit to neutron stars beyond which gravity overwhelms the degeneracy pressure of the neutrons. What do you suppose happens when a neutron star goes above the upper mass limit?