

1. Before Debye's theory of lattice vibrations, there was Einstein's theory. We will explore Einstein's theory in this problem. Assume that you have a solid consisting of N atoms. Assume each atom is a 3D harmonic oscillator so there are $3N$ degrees of freedom. Assume each oscillator has the same set of energy levels, $n\hbar\omega$. Finally assume that the oscillators are not coupled. That is, the each oscillator can oscillate independently of all the other oscillators. You might want to define an "Einstein temperature," $\Theta = \hbar\omega/k$. Determine the energy and heat capacity of this system. Show that for $T \gg \Theta$, the heat capacity agrees with the Dulong and Petit value. Show that for $T \ll \Theta$, the heat capacity goes to zero exponentially. This does not agree with experiment, at least as far as the heat capacity of solids is concerned.

Comment 1. When Einstein made this theory, the low temperature heat capacity data were not very good. He was actually more concerned with explaining why some solids did not obey the Dulong and Petit law at room temperature. The reason is that they had fairly high natural frequencies of oscillation and therefore a large Θ .

Comment 2. Although this theory, based on "uncoupled" oscillators, does not explain the heat capacity of solids, it does explain other things! Whenever we have independent oscillators and the temperature gets below $\hbar\omega/k$, the oscillator is basically in its ground state. Excitations of the oscillator are "frozen out." So, the rotational and vibrational modes of diatomic and polyatomic gases do not contribute to the heat capacities at low temperatures. See K&K figure 3.9. Also, in a liquid, the transverse oscillations of the atoms about their equilibrium positions are basically uncoupled. So at low temperatures, the transverse oscillations freeze out and do not contribute to the heat capacity. This is why in last week's homework problem on the heat capacity of liquid He, we only needed to include the longitudinal modes in the heat capacity.

2. Mathematical exercise as promised in lecture. Determine the chemical potential of an ideal gas by differentiating the energy with respect to particle number at constant volume and entropy.

3. K&K, chapter 5, problem 3.

4. K&K, chapter 5, problem 4.

5. K&K, chapter 5, problem 7.

6. K&K, chapter 5, problem 10.

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7. K&K, chapter 5, problem 12. Note: if you get a very large tree, you have probably solved the wrong problem. A tree doesn't need to hold up a column of water vapor, it needs to hold up a column of water (or rather sap, which is water with some "impurities" dissolved in it). To do this it uses osmotic pressure. Solute molecules in a dilute solution can be treated much the same way we treated an ideal gas. Suppose there is a membrane, through which solvent molecules can flow freely, but through which solute molecules cannot pass. (The membrane can be provided by the cells in the tree.) Also suppose that one side of the membrane contains no solute molecules and the other side contains a concentration n . What is the pressure difference across the membrane? Can you estimate n for sap? Can you estimate the height of a tree?