

1. We can consider blackbody radiation as a gas of photons. Let's calculate the pressure in several ways.

- (a) As noted in lecture, a photon of frequency ω carries energy $\hbar\omega$ and momentum $\hbar\omega/c$. So we have a momentum density (which is 0 when we add up the momentum in all directions) and a momentum flux density. Assuming that the directions of the velocities of the photons in an ideal gas are isotropically distributed, use our method of calculating the momentum flux (see lecture 6) to show that the pressure is

$$p = \frac{u}{3} = \frac{\pi^2 \tau^4}{45 \hbar^3 c^3}.$$

- (b) Use $p = -(\partial U / \partial V)_\sigma$. Recall that changing the volume at constant entropy means keeping the photons in the same states as the energies of the states change. By considering photons in a cubical box of side L , show that at constant entropy,

$$\frac{dV}{V} = 3 \frac{dL}{L} = -3 \frac{d\omega}{\omega} = -3 \frac{d\tau}{\tau}.$$

This result can also be obtained from the expression for the entropy which shows that $\sigma \propto V\tau^3$. Using this result, evaluate the pressure by differentiating the energy with respect to volume at constant entropy and show that you obtain the same expression for the pressure as in part (a). See also K&K, chapter 4, problem 6.

2. By summing the average energy per oscillator in a cavity, we obtained the result that

$$U = \frac{\pi^2}{15 \hbar^3 c^3} V \tau^4.$$

If we divide the average energy per oscillator by the energy per photon in the oscillator, we get the average number of photons in the oscillator. We can then sum this up over all the oscillators to get the total number of photons in the cavity. Carry out this summation and obtain an expression for the number of photons, N . (You will have to look up or numerically evaluate an integral.) Show that the entropy is proportional to the number of photons, $\sigma \approx 3.6N$. See also K&K, chapter 4, problem 1.

3. If we have a cavity and we make a small hole in it, then we will be interested in the energy getting through the hole. Determine an expression for the flux density emerging from the hole or, equivalently, the flux density radiated by a surface which is a perfect blackbody. The flux density is the energy per unit area, per unit time, per unit frequency, per unit solid angle. Note that this will depend on the direction relative to the normal to the surface or the hole. For example, at 90° from the normal, the hole or surface is seen edge on and can't radiate any energy in this direction. Integrate over solid angle and frequency to obtain the total energy per unit area per unit time emitted by a blackbody

surface. This problem is similar in concept to last week's problem on the distortion of the Maxwell velocity distribution for molecules emitted through a small hole. Also, it's similar to K&K, chapter 4, problem 15.

4. K&K, chapter 4, problem 3.
5. K&K, chapter 4, problem 7.
6. K&K, chapter 4, problem 8. See also problem 19.
7. K&K, chapter 4, problem 11.
8. K&K, chapter 4, problem 14.