1. In lecture, we discussed the Maxwell velocity distribution for a low density gas,

$$p(v)\,dv = 4\pi \left(\frac{1}{2\pi\tau/m}\right)^{\frac{3}{2}} e^{-mv^2/2\tau} v^2\,dv\;.$$

(a) Determine the most probable speed (the speed at which the probability density is maximum), the average speed, $\langle v\rangle$, and the root mean square speed, $\sqrt{\langle v^2\rangle}$.

(b) Evaluate these quantities numerically for nitrogen molecules at room temperature (293 K).

2. Problem 1 was kind of plug in and grind. This one requires some thought. One way to examine molecular speeds and compare with the Maxwell distribution would be to heat up a gas in an oven, and let some gas escape to a vacuum through a small hole in the oven. To measure the speed distribution, one can look at the distribution of distance traveled in a fixed time. Determine the distribution of the speeds of molecules which make it through the hole. Assume that the Maxwell distribution applies to the molecules in the oven. Hint: all other things being equal, faster molecules hit the hole more often than slower molecules!

3. Consider the paramagnetic spin system we introduced in lecture 4, and discussed again in lecture 6. Determine the free energy (as a function of τ) for this system. From the free energy, deduce expressions for the energy and entropy. Of course, these expressions should agree with what's been derived before by other techniques. This problem is essentially K&K, chapter 3, problem 1, except that it refers to our model for a paramagnetic system in which the two energy levels of a given magnet occur at $-E$ and $+E$.

4. K&K, Chapter 3, Problem 9. Show that the partition function of two independent systems in thermal contact, but weakly interacting, is just the product of the partition functions of the two systems:

$$Z_{1\,\mathrm{and}\,2}(\tau) = Z_1(\tau)\cdot Z_2(\tau)\;.$$

5. K&K, Chapter 3, Problem 7. It's amazing that such a simple model might actually be relevant to something as complex as DNA!

6. K&K, Chapter 3, Problem 10. Note also, warming a gas makes it expand. Expanding a gas makes it cool. What happens to a rubber band when it is expanded? Your upper lip is a good temperature sensor. Place a rubber band in contact and stretch it.(Be careful, overdoing it could hurt!) Does it get hotter or colder?

7. K&K, Chapter 3, Problem 11.

8. In lecture 6, pages 1–3, we found expressions for probabilities that maximized the entropy. In one case, we found

$$p_i = e^{-1 - \lambda_1 - \lambda_2 E_i} ,$$

where E_i is the energy of state i and λ_1 and λ_2 are Lagrange multipliers. Show that $\lambda_2 = 1/\tau$. Hint: if we make changes δp_i to the probabilities, what must $\sum \delta p_i$ satisfy? What are $\delta\sigma$ and δU?