1. Consider a system in which the particles are Physics 301 students confined to room A06. Suppose the particles have no kinetic energy so they occupy chairs in the room and there is no interaction energy so there is at most one particle per chair. What is the entropy of this system?

2. In lecture 1, we showed that the number of states for a mole of gas in a container of volume $2V$ divided by the number of states for the same gas in a container of half the volume (V), is roughly $10^{N_0/3}$ where $N_0 = 6 \times 10^{23}$ is Avogadro's number. Assume you have a mole of air (N_2) at STP. In units of the age of the universe, how long would you have to wait for the air to wind up in half its container (i.e. that there are even odds that that's happened)? The age of the universe is roughly 5×10^{17} s and a typical speed of an air molecule is roughly 5×10^4 cm/s.

3. This problem is a generalization of K&K problem 1 in chapter 2. Suppose the number of microstates (or multiplicity function) for a system with energy U, volume V, and number of particles N is

$$g(U, N, V) = CU^{\alpha N} V^{\beta N} ,$$

where C, α, and β are constants.

(a) What are the temperature and pressure of this system?

(b) If the system is an ideal gas, what can you say about α and β? In particular, can you relate α to the specific heat at constant volume? What must β be in order to recover the ideal gas law?

4. This is essentially K&K, chapter 2, problem 2 and concerns the paramagnetic spin system. In this problem we assume that the temperature τ is large compared with the magnetic energy mB, so the spin excess $2s$ is small compared with the total number of spins N. Then the entropy as a function of spin excess is

$$\sigma(s) \approx \sigma_0 - 2s^2/N ,$$

where $\sigma_0 = \sigma(0) = \log g(N, 0)$. If U is the internal energy if the system, show that

$$\sigma(U) = \sigma_0 - \frac{U^2}{2m^2B^2N} ,$$

$$\frac{1}{\tau} = -\frac{U}{m^2B^2N} ,$$

and finally, find the equilibrium fractional magnetization: $M/mN = 2\langle s\rangle/N$.

Comment: note that $U < 0$ since the aligned magnetic dipoles have negative energy $-E = -mB$ and anti-aligned dipoles have positive energy $E = mB$. We can rearrange the above expression as

$$-\frac{U}{N}\tau = E^2 ,$$

which says that the average energy per dipole U/N times the energy scale set by the temperature, τ, is a constant $(mB)^2$. The system would like to reduce its energy (make U as negative as possible) but it also wants to increase its entropy (make the alignment as random as possible). The temperature controls which is the more important. At high temperatures, maximizing entropy wins and $U \to 0$. At low temperatures, minimizing energy wins and U gets large and negative. (The above expression goes to $-\infty$, but that's a sign of the approximation breaking down!)

5. K&K, This comes from K&K, chapter 2, problem 3. A quantum harmonic oscillator with frequency ω can exist in states with energies (relative to to the zero point energy) of $\hbar\omega$, $2\hbar\omega$, $3\hbar\omega$, ... If there are N identical oscillators, the number of ways to obtain an energy $n\hbar\omega$ is

$$g(N, n) = \frac{(N + n - 1)!}{n!(N - 1)!} \; .$$

(This is worked out at the end of K&K chapter 1.)

(a) Find the entropy of this system of oscillators when N is large (so that you can use the Stirling approximation, $\log N! \approx N \log N - N$ and you can ignore 1 compared to N.

(b) Write the entropy as a function of the total energy $U = n\hbar\omega$ and the number of oscillators N: $\sigma = \sigma(U, N)$. Use this to find the expression for the energy in terms of the temperature:

$$U = \frac{N\hbar\omega}{\exp(\hbar\omega/\tau) - 1} \; .$$

6. The Poisson Distribution. We discussed the binomial distribution in class. Recall, the probability of obtaining n up spins with N systems is

$$p_n = \binom{N}{n} p^n (1 - p)^{N-n} \; ,$$

where p is the probability that any one system has an up spin. Consider the binomial distribution in the limit that $N \to \infty$, $p \to 0$, but $Np \to r$.

(a) Show that the probability of obtaining n is

$$p_n = \frac{r^n}{n!} e^{-r} \; .$$

This is called the Poisson distribution. This distribution occurs in counting. For example, counting nuclear decays. Can you see why this is applicable? (Consider a time interval in which, on the average, there will be r decays and divide the interval into a large number, N, of equal subintervals. Then the probability of a decay in any interval is r/N, and if N is large enough you don't need to worry about two or more decays in an interval...)

(b) Show that the mean and variance of the Poisson distribution are both r. What is the fractional uncertainty?

Note: you should of course look at the other problems in K&K. You will probably find chapter 2, problem 4 amusing!