PHYSICS DEPARTMENT, PRINCETON UNIVERSITY

PHYSICS 301 FINAL EXAMINATION

January 21, 2004, 1:30–4:30pm, Jadwin A07

This exam contains five problems. Work any three of the five problems. All problems count equally although some are harder than others. Do all the work you want graded in the separate exam books. *Indicate clearly which three problems you have worked and want graded.* I will only grade three problems. If you hand in more than three problems without indicating which three are to be graded, I will grade the first three, only!

Write legibly. If I can't read it, it doesn't count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

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Physical Constants and Conversion Factors

$$
\begin{aligned}
c &= 2.998 \times 10^{10}\ \mathrm{cm\,s^{-1}}\,, \\
\hbar &= 1.054 \times 10^{-27}\ \mathrm{erg\,s}\,, \\
k &= 1.380 \times 10^{-16}\ \mathrm{erg\,K^{-1}}\,, \\
e &= 4.803 \times 10^{-10}\ \mathrm{statcoulomb}\,, \\
N_0 &= 6.025 \times 10^{23}\ \mathrm{molecules\ \ mole^{-1}}\,, \\
m_{\mathrm{electron}} &= 9.108 \times 10^{-28}\ \mathrm{g}\,, \\
m_{\mathrm{proton}} &= 1.672 \times 10^{-24}\ \mathrm{g}\,, \\
m_{\mathrm{neutron}} &= 1.675 \times 10^{-24}\ \mathrm{g}\,, \\
m_{\mathrm{amu}} &= 1.660 \times 10^{-24}\ \mathrm{g}\,, \\
\mu_B &= 9.273 \times 10^{-21}\ \mathrm{erg\ \ Gauss^{-1}}\,, \\
G &= 6.673 \times 10^{-8}\ \mathrm{cm^3\,s^{-2}\ \ g^{-1}}\,.
\end{aligned}
$$

$$
\begin{aligned}
1\ \mathrm{atm} &= 1.013 \times 10^{6}\ \mathrm{dyne\,cm^{-2}}\,, \\
1\ \mathrm{eV} &= 1.602 \times 10^{-12}\ \mathrm{erg}\,, \\
1\ \mathrm{cal} &= 4.186 \times 10^{7}\ \mathrm{erg}\,.
\end{aligned}
$$

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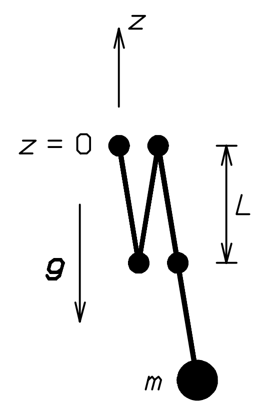
1. Consider a polymer consisting of N links (like links of a chain) of length L. The links are massless. One end of the polymer is fixed at height $z = 0$. The other end is attached to a mass m. Each link can pivot about its attachment with the previous link or the fixed end so that a link may carry the polymer "up" or "down" a distance L. For example, the figure shows a polymer containing four links, taking the polymer down, up, down, and down. The figure is schematic; all links are actually vertical. The mass at the end of the polymer is in a uniform gravitational field $\boldsymbol{g}$, so it has a gravitational potential energy mgz. The polymer is in weak thermal contact with a reservoir at temperature τ. For simplicity, suppose N is even and suppose the only energy in the system is the gravitational potential energy of the mass.



(a) Without doing any calculations, what is the gravitational potential energy of the system when it is very cold ($\tau \ll mgL$)? Be sure and give a brief explanation.

(b) Again, without any calculations, what is the gravitational potential energy of the system when it is very hot ($\tau \gg NmgL$)? Again, a brief explanation is desired.

(c) What is the partition function of the system?

(d) What is the gravitational potential energy of the system at temperature τ?

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2. Suppose that we lived in a world with two (instead of three) spatial dimensions but physics was otherwise the same (hard to imagine ...). Consider a low density gas of non-interacting point particles in this world. The gas has N particles of mass m, occupies a volume (area) A and is in equilibrium at temperature τ.

(a) Determine the partition function of this gas in the limit that multiply occupied states may be ignored. Hint: helpful expressions for this part and subsequent parts:

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} e^{-u^2/2}du = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} u^2 e^{-u^2/2}du = 1\,, \qquad \log N! \approx N\log N - N\,.$$

(b) Determine the energy and the (Helmholtz) free energy of this gas.

(c) Determine the pressure (a force per unit length in the two dimensional world) and the entropy of this gas.

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3. This problem contains two "quickies" each of which will receive equal weight in the grading.

(a) Consider a ferromagnetic solid containing N spins (with magnetic moments) each of which may point up or down. Above the Curie temperature, τ_c, the spins are completely randomly oriented. Below the Curie temperature, the spins spontaneously align becoming completely aligned (either all up or all down) as $\tau \to 0$. Suppose it is found that a reasonable approximation to the heat capacity (of the spin system alone) as a function of temperature is

$$C(\tau) = \begin{cases} C_c(\tau^2/\tau_c^2), & \tau < \tau_c, \\ 0, & \tau > \tau_c, \end{cases}$$

where C_c is a constant. By considering the entropy at $\tau = 0$ and at $\tau = \tau_c$, determine the value of the constant C_c.

(b) Do you find it surprising that life can exist thousands of meters below the surface of the ocean where the pressures are huge and no sunlight can penetrate? The energy source is the heat from hydro-thermal vents in the ocean floor. These are places where molten rock is close to the ocean floor and can heat the water. In this problem, you're asked to estimate the boiling point (temperature) of water at these depths. For definiteness, take the density of sea water to be $\rho = 1$ g cm^{-3} (even though salt water is somewhat denser!), the depth to be $D = 2$ km $= 2 \times 10^5$ cm, the gravitational acceleration to be $g = 980$ cm s^{-2}, the latent heat of vaporization of water to be $\ell = 2.3 \times 10^{10}$ ergs g^{-1}, the specific volume of water vapor at the normal boiling point (373 K, 1 atm) to be $v_v = 1700$ cm^3 g^{-1}, and the molar mass of water to be 18 g mole^{-1}. Please express your answer for the temperature in Kelvin, not ergs! Finally, remember that *estimate* means to make reasonable approximations.

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4. In this problem we consider a toy model for a white dwarf star. We assume that the star is spherically symmetric and has uniform density (the main reason it's a toy model), that the star is cold, that the only effects to be considered are the degeneracy pressure of the electrons and gravitational energy. Assume that the star is made of fully ionised helium so there are two electrons for every four nucleons. Assume the star has mass M and radius R. Your solutions will probably involve the electrom mass m_e and the nucleon (proton or neutron, they're the same for this problem) mass, m_n. Altogether there are three masses: M, m_e, and m_n. Please be clear about which mass is which!

(a) What is the gravitational potential energy of the star? Express your answer in terms of M, R, and whatever physical constants you need.

(b) What is the kinetic energy of the electron system? Express your answer in terms of M and R and whatever physical constants you need. (Remember that electrons have spin 1/2 so there are two electrons per state.)

(c) What is the equilibrium radius of the white dwarf as a function of M and whatever physical constants you need?

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5. Consider a uniform sphere of radius R. There is a uniformly distributed energy source throughout the volume of the sphere (for example, from uniformly distributed radioactive elements). The energy generation rate per unit volume is ϵ. The dimensions of ϵ are power per unit volume. The sphere is floating in space and is immersed in the cosmic backround radiation at temperature $T_0 = \tau_0/k$, where k is the Boltzmann constant. The sphere is a perfect absorber and emitter (i.e. a perfect blackbody). The thermal conductivity of the material in the sphere is K and all heat transfer internal to the sphere is by conduction . Finally, the sphere has heat capacity per unit volume, C.

(a) Assuming the sphere has reached a steady state, what is the surface temperature, τ_s of the sphere?

(b) Assuming the sphere has reached a steady state, what is the temperature as a function of radius, r, inside the sphere? You may use τ_s for the surface temperature rather than substitute the expression you derived in part (a).

(c) Suppose the energy source has been off for a long time and the temperature of the sphere is uniform at τ_0. The energy source is turned on. Estimate (crudely is fine) how long it will take for the sphere to reach a steady state. You may assume $\tau_s \gg \tau_0$.

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