

PHYSICS DEPARTMENT, PRINCETON UNIVERSITY

PHYSICS 301 FINAL EXAMINATION

January 13, 2005, 7:30–10:30pm, Jadwin A10

This exam contains five problems. Work any three of the five problems. All problems count equally although some are harder than others. Do all the work you want graded in the separate exam books. *Indicate clearly which three problems you have worked and want graded.* I will only grade three problems. If you hand in more than three problems without indicating which three are to be graded, I will grade the first three, only!

Write legibly. If I can't read it, it doesn't count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

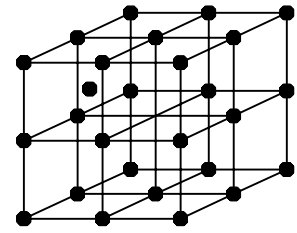
Physical Constants and Conversion Factors

$$\begin{aligned}c &= 2.998 \times 10^{10} \text{ cm s}^{-1}, \\ \hbar &= 1.054 \times 10^{-27} \text{ erg s}, \\ k &= 1.380 \times 10^{-16} \text{ erg K}^{-1}, \\ e &= 4.803 \times 10^{-10} \text{ statcoulomb}, \\ N_0 &= 6.025 \times 10^{23} \text{ molecules mole}^{-1}, \\ m_{\text{electron}} &= 9.108 \times 10^{-28} \text{ g}, \\ m_{\text{proton}} &= 1.672 \times 10^{-24} \text{ g}, \\ m_{\text{neutron}} &= 1.675 \times 10^{-24} \text{ g}, \\ m_{\text{amu}} &= 1.660 \times 10^{-24} \text{ g}, \\ \mu_B &= 9.273 \times 10^{-21} \text{ erg Gauss}^{-1}, \\ G &= 6.673 \times 10^{-8} \text{ cm}^3 \text{ s}^{-2} \text{ g}^{-1}.\end{aligned}$$

$$\begin{aligned}1 \text{ atm} &= 1.013 \times 10^6 \text{ dyne cm}^{-2}, \\ 1 \text{ eV} &= 1.602 \times 10^{-12} \text{ erg}, \\ 1 \text{ cal} &= 4.186 \times 10^7 \text{ erg}.\end{aligned}$$

1. Two identical objects, A and B, are thermally and mechanically isolated from the rest of the world. Their initial temperatures are $\tau_A > \tau_B$. Each object has heat capacity C (the same for both objects) which is independent of temperature.
 - (a) Suppose the objects are placed in thermal contact and allowed to come to thermal equilibrium. What is their final temperature? How much entropy is created in this process? How much work is done on the outside world in this process?
 - (b) Instead, suppose objects A (temperature τ_A) and B (temperature $\tau_B < \tau_A$) are used as the high and low temperature heat reservoirs of a heat engine. The engine extracts energy from object A (lowering its temperature), does work on the outside world, and dumps waste heat to object B (raising its temperature). When the temperatures of A and B are the same, the heat engine is in the same state as it started and the process is finished. Suppose this heat engine is the most efficient heat engine possible. In other words, it performs the maximum work possible. What is the final temperature of the objects? How much entropy is created in this process? How much work is done on the outside world in this process?

2. A cubic crystal contains N atoms. The atoms can exist at lattice sites or an atom may find itself displaced from its normal site into the center of one of the 8 adjacent unit cells as suggested by the figure. When an atom is displaced, its energy is increased by $\epsilon > 0$ over its energy at the normal lattice site. Suppose the crystal is in equilibrium at temperature τ . Assume $\tau \ll \epsilon$ so the chance that two atoms try to occupy the same cell is negligible. Calculate the partition function, the entropy, the energy (relative to the energy the crystal would have if all atoms were at their normal sites), and the number of atoms in displaced sites.



3. Consider a spinless particle of mass m bound to the origin by a quadratic potential $V = k(x^2 + y^2 + z^2)/2 - 3\hbar\omega/2$. In other words, a three dimensional harmonic oscillator. Its energies are $\epsilon(n_x, n_y, n_z) = (n_x + n_y + n_z)\hbar\omega$. (The zero point energy was removed in the expression for the potential!) The frequency $\omega^2 = k/m$ is the natural frequency of the oscillations and n_x , n_y , and n_z are all integers greater than or equal to zero.

Some things to note: All energies are an integer times $\hbar\omega$ and the degeneracy increases as the energy increases. The energy of the ground state is 0 ($n_x = n_y = n_z = 0$) and the ground state is non-degenerate.

- (a) Obtain an expression for $\mathcal{D}(\epsilon)$, the density of states, valid at energies much larger than $\hbar\omega$. (Recall that the number of states with energies less than ϵ is $\int_0^\epsilon \mathcal{D}(\epsilon') d\epsilon'$.) Hint: try a geometric approach and note that the energy is a linear function of the quantum numbers.
- (b) Now suppose there are N (N is very large, the order of Avogadro's number, say) noninteracting identical particles in the harmonic oscillator potential just described. The particles are in equilibrium at temperature τ which is much less than $\hbar\omega$ so that almost all particles are in the ground state. (Remember, they're spinless, so they're bosons and there's no limit to how many particles may occupy a state!) What is the chemical potential in this low temperature limit?
- (c) This system has something like a Bose Einstein Condensation (BEC) such that the number in the ground state is large even for temperatures comparable to $\hbar\omega$. The number of particles in the ground state is

$$N_0 = N \left(1 - \left(\frac{\tau}{\tau_E} \right)^\alpha \right),$$

where τ_E is the "Einstein condensation temperature." Determine a numerical value for α and an expression for τ_E . In deriving this expression, you may encounter an intractable integral. It is not necessary to evaluate this integral. Instead convert it to dimensionless form (that is, it does not contain \hbar , ω , τ , μ or N) and call its value I . Make suitable approximations!

4. A small spherical particle, radius $r = 1\mu = 10^{-4}$ cm moves in room temperature ($T = 300$ K) water. The density of the particle is about the same as that of water, $\rho = 1$ g/cm³, and the viscosity of water is about $\eta = 0.01$ poise = 0.01 g/s cm.

In all cases below, both an analytic expression and a numerical estimate are desired.

- (a) Suppose the particle is moving with some initial velocity. Use dimensional analysis and what you know about viscosity to estimate the decay time for the particle's velocity.
- (b) Assuming the particle is in equilibrium at temperature T , what is its typical velocity?
- (c) About how far will the particle travel in one decay time?
- (d) About how far will the particle travel in $M = 1$ million decay times? (Note: this question is asking for the typical distance between the particle's position at some time and its position a million decay times later.)

5. During our studies of thermal physics, we've often considered an ideal gas. Sometimes we needed a model for departures from an ideal gas and we used a van der Waals gas. Of course, this isn't a perfect description of a real gas either, and lots of folks have invented modifications to the ideal gas equation of state that describe departures from ideal gas behavior. Here's Dieterici's equation of state for N molecules of gas occupying a volume V at pressure p and temperature τ ,

$$p(V - Nb) = N\tau e^{-Na/\tau V} ,$$

where a and b are constants (not necessarily the same constants that occur in the van der Waals equation of state).

- (a) Give brief statements of the physical significance of the constants a and b .
- (b) Construct a p - V diagram and sketch some representative isotherms for this gas. Be sure to sketch the critical isotherm, at least one isotherm at a higher temperature than the critical isotherm and at least one isotherm at a lower temperature than the critical isotherm. Be sure to label the axes and the curves appropriately. Hints: you may want to use dimensionless variables—if so, be sure to specify the relation between your variables and p , V , and τ . You may also want to consider part (c) while you do this part.
- (c) Determine p_c , V_c , and τ_c , that is, the pressure, volume, and temperature at the critical point.

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