Please BOX your answers.

\[ h = 6.63 \times 10^{-34} \text{ J s} \quad hc = 1240 \text{ eV nm} \quad hc/m_e c^2 = 2.43 \times 10^{-12} \text{ m} \quad e = 1.60 \times 10^{-19} \text{ C} \]

1. As you know, an electron bound in a hydrogen atom, in its ground state, has (electric) potential energy \( U = -27.2 \text{ eV} \), kinetic energy \( K = 13.6 \text{ eV} \) and total energy \( E = -13.6 \text{ eV} \). In this problem we are going to consider an electron in a one dimensional box of length \( L \) as a “toy model” for a hydrogen atom, note that the electron’s position will be given by the coordinate \( x \) with \( 0 \leq x \leq L \).

(a) What are the wave functions and associated kinetic energies for the electron confined to this box. (This problem is asking for formulae, not numbers.) (3 points)

(b) What is the size of the box (in nm) such that the electron in the lowest kinetic energy state has the same kinetic energy as the the electron in the hydrogen atom ground state? (A numerical value is desired.) (2 points)

Rewrite and sign the Honor Pledge: I pledge my honor that I have not violated the Honor Code during this examination.

Signature

(OVER—this problem continues on the next page.)
(c) Now suppose the we put two of these “box atoms” right next to each other, end-to-end, so the electron moves in a box with \(0 \leq x \leq 2L\). That is, the box is twice as long. Now what is the ground state kinetic energy of the electron? (2 points)

(d) If each box is a “toy model” of a hydrogen atom, then two boxes right next to each other must be a toy model of a hydrogen molecule! \((\text{H}_2)\) However, a hydrogen molecule has two electrons. Ignoring interactions between the electrons, what is the ground state kinetic energy of two electrons in this double length box. (2 points)

(e) Recall that electrons have spin \(\hbar/2\). What can you say about the directions of the spins of the electrons in the “hydrogen molecule” of part (d)? Are they parallel or anti-parallel? (1 point)