SOLUTIONS

Quiz Number 4

Please BOX your answers.

\[ c = 3 \times 10^8 \text{ m/s} \quad 1/4\pi\epsilon_0 = 9 \times 10^9 \text{ m/F} \quad \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2. \]

1. The electric field of an electromagnetic wave propagating in the \( x \) direction is \( E = E_0 k \sin(kx - \omega t) \), where \( E_0 = 3 \text{ V/m} \) and \( k = 20\pi \text{ rad/m} \).

(a) Give an expression for the magnetic field of this wave. Be sure your expression specifies the direction of the magnetic field and gives a numerical value for the strength of the field. (3 points)

\[ E \times B \] must point in the direction of propagation, the \( x \) direction. This means \( B \) points in the negative \( y \) direction. Also, \( B_0 = E_0/c \). So \( B = -B_0 \hat{j} \sin(kx - \omega t) \) with \( B_0 = E_0/c = 10^{-8} \text{ T} \).

(b) What is the wavelength of this wave? (1 point)

\[ \lambda = 2\pi/k = 0.1 \text{ m}. \]

(c) What is the frequency (Hz) of this wave? (1 point)

\[ c = c/\lambda = 3 \text{ GHz}. \] Also \( \omega = 6\pi \times 10^9 \text{ rad/s} \).

(d) What is the average intensity of this wave? (2 points)

\[ \langle S \rangle = E_0 B_0/2\mu_0 = 0.012 \text{ W/m}^2. \]

Rewrite and sign the Honor Pledge: I pledge my honor that I have not violated the Honor Code during this examination.
2. A plane electromagnetic wave of frequency $\omega$ and magnetic field amplitude $B_0$ is detected by means of a small coil of wire oriented perpendicular to the direction of the magnetic field in the wave. The coil is much smaller than the wavelength of the wave, so the magnetic field through the coil can be taken to be constant over the cross section of the coil and sinusoidally varying in time, $B = B_0 \sin \omega t$. The cross section of the coil encloses area $A$ and the coil contains $N$ turns. What is the emf in the coil resulting from the wave? (3 points)

We can take the field at the coil to be $B_0 \sin \omega t$. Then the flux through a single turn of the coil is $B_0 A \sin \omega t$ and through all turns it’s $NB_0 A \sin \omega t$. The emf is just the negative rate of change of the flux, so $\mathcal{E} = -\omega NB_0 A \cos \omega t$. 