1. Consider an α decay of a heavy nucleus (to be definite, suppose $212 \leq A \leq 256$). The nucleus is originally at rest. The energy release is small compared to the rest mass energies of the particles involved, so the daughter nucleus and the alpha particle can be considered non-relativistic and treated with Newtonian mechanics.

(a) Using conservation of energy and momentum, estimate (to one significant digit) the fraction of the kinetic energy carried by the daughter nucleus. (2 points)

Since we have two particles and total momentum 0 (the decaying particle was at rest), conservation of momentum tells us that the α particle and the daughter nucleus have equal and opposite momenta. Let the magnitude of the momentum be $p$. Then the kinetic energy of the daughter nucleus is $E_{\text{daughter}} = p^2/2m_{\text{daughter}} \approx p^2/(A - 4)m_p$ and the energy of the α particle is $E_{\alpha} = p^2/2m_\alpha \approx p^2/2 \cdot 4m_p$. The total is the sum of the two. The fraction carried by the daughter nucleus is $p^2/(A - 4)m_p/(p^2/(A - 4)m_p + p^2/2 \cdot 4m_p) = 4/A = \frac{2}{A}$. With $A = 212$, 1.89%, with $A = 256$, 1.56%. To one digit, these are both 2%!

(b) Your answer from part (a) should show that it’s a good approximation to assume that the daughter nucleus is at rest and the α particle carries all the kinetic energy. Consider the decay of thorium: $^{232}\text{Th} \rightarrow ^{228}\text{Ra} + ^4\text{He}$. The masses of the atoms are $m[^{232}\text{Th}]c^2 = 216127.9\text{ MeV}$, $m[^{228}\text{Ra}]c^2 = 212395.7\text{ MeV}$, and $m[^4\text{He}]c^2 = 3728.2\text{ MeV}$. What is the speed (in units of $c$) of the alpha particle? (4 points)

First, we figure out the energy release as the difference in the masses of the original and final particles: $E = \Delta mc^2 = m[^{232}\text{Th}]c^2 - (m[^{228}\text{Ra}]c^2 + m[^4\text{He}]c^2) = 4.0\text{ MeV}$. We equate this to the kinetic energy of the alpha particle: $E = (1/2)m[^4\text{He}]v^2 = (1/2)m[^4\text{He}]c^2(v/c)^2$. So, $v/c = \sqrt{2E/m[^4\text{He}]c^2} = \sqrt{2 \cdot 4.0/3728.2} = 0.046$.

Rewrite and sign the Honor Pledge: I pledge my honor that I have not violated the Honor Code during this examination.

Signature

(OVER)
2. As we saw in lecture, technetium-99 is used as a radioactive tracer in such applications as bone scans. Technetium in an excited state is attached to a drug or chemical taken up by the tissue under study and given to the person to be tested. The excited technetium decays with a 6 hour half life to its ground state emitting a 142 keV gamma ray and the scan is a “photograph” taken with these gamma rays. Technetium in its ground state is unstable and beta decays according to

\[ ^{99}\text{Tc} \rightarrow ^{99}\text{Ru} + e^- + \bar{\nu} \]

releasing 0.29 MeV of binding energy. This decay has a half life of 210,000 years. In this problem we want to make a conservative estimate of the radiation dose rate (in rem/year) caused by this ground state technetium in a patient’s body. This estimate will be conservative in that all the uncertainties will be in the direction of overestimating the dose. We assume that the technetium stays in the body (not true, it is eliminated with a “biological half life”). We assume that all the beta particles resulting from the decay are absorbed in the body (not true, some escape). We assume that the beta particles get all the released energy (not true, the neutrinos get some of the energy). Assume that \( N_0 = 2 \times 10^{13} \) atoms (about 3 nanograms) of \(^{99}\text{Tc}\) are injected into an 80 kg patient. Recall that the RBE of \( \beta \) particles is about 1. What is the average dosage rate in rem per year that this patient would receive over the first half-life? (Assume the patient’s body holds together for 210,000 years!) (4 points).

In the first half-life, half the atoms decay. So there are \( N = 10^{13} \) decays, each releasing 0.29 MeV \( \cdot 1.6 \times 10^{-13} \) J/MeV = \( 4.64 \times 10^{-14} \) J. The total dose in Grays is the energy divided by the mass in which it is deposited. To convert to rads, we must multiply by 100, and to convert to rems we must multiply by the RBE which is 1. Thus the dose in the first half-life is

\[
100 \cdot 1 \cdot 4.64 \times 10^{-14} \text{ J} \cdot 10^{13} / 80 \text{ kg} = 0.58 \text{ rem}.
\]

To convert to a dose rate, we divide by the half-life, 210,000 yr and get \( 0.0028 \text{ mrem/yr} \). Note that the actual rate while the patient is alive will be slightly larger than this because there are more decays per year at the beginning of the 210,000 years than the average over the first half-life. With a little arithmetic, we can calculate that we need to multiply the above dosage rate by \( 2 \ln 2 = 1.4 \) to account for this. (But this wasn’t part of the problem!) The number of technetium atoms given in the problem is based on a dose of 20 mCi found on the web in instructions for a bone scan.