1. Rainbow: a ray of sunlight enters a spherical droplet at an angle of incidence $\theta_1 = 55^\circ$ measured with respect to the normal to the surface. It is reflected from the back surface of the droplet and reenters into the air. Index of refraction for water is $n = 1.33$.

(a) What is the angle of incidence $\theta_2$ measured with respect to the normal to the surface when the ray enters the droplet? [2 points]

\[ \frac{n_1 \sin \theta_1}{n_w} = \frac{n_2 \sin \theta_2}{n_w} \]

\[ \sin \theta_2 = \frac{\sin \theta_1}{n_w} = \frac{\sin 55^\circ}{1.33} \]

\[ \theta_2 \approx 36^\circ \]

(b) What is the angle $\alpha$ between the incoming and outgoing ray (Hint: consider isosceles triangles formed by the center of the droplet, the point of refraction and the point of reflection). [2 points]

\[ \lambda + \vartheta_1 + \vartheta_1 + 180^\circ - 2 \vartheta_2 + 180^\circ - 2 \vartheta_2 = 360^\circ \]

\[ \lambda = 4 \vartheta_2 - 2 \vartheta_1 \]

\[ \lambda = 42^\circ \]

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REWRITE AND SIGN THE HONOR PLEDGE: “I pledge my honor that I have not violated the Honor Code during this examination”

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Signature
(c) Assume index of refraction for red color is \( n = 1.33 \) and for blue color \( n = 1.34 \). What is the difference in angle between the outgoing rays of the two colors assuming they both enter the droplet at the same angle \( \theta_1 \) (this gives an estimate of the angular width of the rainbow)? [1 point]

\[
\theta_2^{\text{Blue}} = \frac{\sin \theta_1}{n_{\text{Blue}}} = \frac{\sin 5^\circ}{1.34}
\]

\[
\Delta \theta = \theta - \theta_2^{\text{Blue}} = 4 \left( 38^\circ - 37.7^\circ \right) = 1.2^\circ
\]

2. A 2 cm coin is placed 8 cm to the left of a converging lens with a focal length 5 cm.

(a) Draw the ray diagram to show the position of the image. What is the image distance from the lens? [2 points]

\[
\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}
\]

\[
d_i = \frac{f \cdot d_o}{d_o - f} = \frac{5 \times 8 \text{ cm}}{3} = 13.33 \text{ cm}
\]

\[
h_i = -\frac{d_i}{d_o} = \frac{-13.33 \text{ cm}}{6} = -2.22 \text{ cm}
\]

(b) A second converging lens with focal length 10 cm is placed at a distance 20 cm to the right of the first lens. Find the size of the coin image for the combined system. [3 points]

\[
L = 20 \text{ cm}
\]

\[
d_i' = 13.33 \text{ cm}
\]

\[
d_o' = 6.67 \text{ cm}
\]

\[
\frac{1}{L^2} = \frac{1}{d_o'^2} + \frac{1}{d_i'^2}
\]

\[
d_i'^2 = \frac{f \cdot d_o'}{d_o' - f} = \frac{10 \text{ cm} \cdot 6.67 \text{ cm}}{(6.67 - 10) \text{ cm}}
\]

\[
= -2.65 \text{ cm}
\]

\[
h_i' = -\frac{h_i}{d_o'} = -\frac{3.33 \text{ cm} \cdot 2.0 \text{ cm}}{6.67 \text{ cm}} = -10 \text{ cm}
\]
(c) Assume index of refraction for red color is \( n = 1.33 \) and for blue color \( n = 1.34 \). What is the difference in angle between the outgoing rays of the two colors assuming they both enter the droplet at the same angle \( \theta_1 \) (this gives an estimate of the angular width of the rainbow)? [1 point]

\[
\begin{align*}
\theta_2^{\text{blue}} &= \sin^{-1} \frac{\sin \theta_1}{n^{\text{blue}}} = \sin^{-1} \frac{\sin 55^\circ}{1.34} \\
\theta_2^{\text{blue}} &= 40.18^\circ \\
\Delta \theta &= \theta_2^{\text{blue}} - 2 \theta_1 = 4(38^\circ - 37.7^\circ) = 1.2^\circ
\end{align*}
\]

2. A 2 cm coin is placed 8 cm to the left of a converging lens with a focal length 5 cm.

(a) Draw the ray diagram that shows the position of the image. What is the image distance from the lens? [2 points]

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \\
d_i = \frac{f \cdot d_o}{d_o - f} = \frac{5 \cdot 8 \text{cm}}{3 \text{cm}} = \frac{13,33 \text{ cm}}{3} \\
l_i = -\frac{d_i}{d_o} \\
l_o = -\frac{13,33 \text{ cm}}{6} = -2,22 \text{ cm}
\]

(b) A second converging lens with focal length 10 cm is placed at a distance 20 cm to the right of the first lens. Find the size of the coin image for the combined system. [3 points]

\[
\begin{align*}
L &= 20 \text{ cm} \\
d_i' &= 13,33 \text{ cm} \\
d_o' &= 6,67 \text{ cm} \\
\frac{1}{f^2} &= \frac{1}{d_o^2} + \frac{1}{d_i^2} \\
d_i^2 &= \frac{f^2 \cdot d_o}{d_o^2 - f^2} = \frac{10 \text{ cm} \cdot 6,67 \text{ cm}}{(6,67 - 10) \text{ cm}} \\
&= -20 \text{ cm} \\
h_i^2 &= -\frac{h_i'}{d_o'} - \frac{20 \text{ cm}}{6,67 \text{ cm}} = -10 \text{ cm}
\end{align*}
\]