1. For each of the three figures below (electrostatic charge distribution, permanent magnets, and currents), indicate whether the net force between the two bars is attractive or repulsive. (3 points)

a) [Diagram of three positive charges]  b) [Diagram of two neutrons]  c) [Diagram of two oppositely charged currents]

2. A P102 student finds that her television reception varies as she moves around her room. Suggest an explanation for this, and roughly estimate the typical distance she needs to move to change the reception from poor to good, if the TV station is broadcasting at 80 MHz. (3 points)

A possible explanation is that there is interference between electromagnetic waves going directly to the TV and those reflecting from the student.

The wavelength is \( \lambda = \frac{C}{f} = 3.75 \text{ m} \), so typically she needs to move

\[ \frac{1}{2} \lambda \approx \frac{3.75}{2} \text{ m} \approx 1.9 \text{ m} \]

3. Two glass plates 10 cm long are in contact at one end and separated by a piece of paper with thickness \( d = 0.02 \text{ mm} \) at the other end. When the plates are illuminated from above with a monochromatic light source (\( \lambda = 600 \text{ nm} \)), dark fringes appear. What is the horizontal spacing between them? (4 points)

Light reflects from the top surfaces of the plates. The 2 reflectors, 1 & 2, interfere.

For dark fringes:

\[ b_{m} = \frac{2 \times 10^{-5} \text{ m}}{1 \times 10^{-1} \text{ m}} = 0.0002 \text{ m} \]

Destructive interference:

\[ 2b_{m} = (m + \frac{1}{2}) \lambda \]

The thickness \( d \) varies as one moves along the plates. To go between 2 dark fringes:

\[ 2 \Delta t = 2b_{m} \quad \Rightarrow \quad \Delta t = \frac{b_{m}}{2} \]

\[ d = \frac{\Delta t}{b_{m}} = \frac{\lambda}{b_{m}} = 0.0015 \text{ m} = 1.5 \text{ mm} \]
4. This problem is about the Bohr model of the hydrogen atom. Assuming that the angular momentum $L$ is quantized, Bohr calculated that the radius of the electron’s circular orbit is given by $r_n = (5.29 \times 10^{-11} \text{ m}) n^2$, where $n = 1, 2, \ldots$. In the following all your answers should be given as a function of $n$.

a. Assume the electron is held in its orbit by electrostatic forces. Show that the speed of the electron is $v_n = n^{-1} \times (2.19 \times 10^6 \text{ m/s})$. (2 points)

$$F = \frac{ke^2}{r^2} = ma = \frac{mv^2}{r} \Rightarrow v_n = \sqrt{\frac{ke^2}{mr_n}} = \frac{1}{n} \left( \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{(9.11 \times 10^{-31})(5.29 \times 10^{-11})} \right)^{\frac{1}{2}} = \frac{1}{n} (2.19 \times 10^6 \text{ m/s})$$

b. Calculate the angular momentum of the electron and explicitly show that it is quantized in units of $\hbar/2\pi$. (2 points)

$$L_n = m v_n r_n = \frac{(9.11 \times 10^{-31})}{n} (2.19 \times 10^6)(5.29 \times 10^{-11}) n^2 = (1.05 \times 10^{-34} \text{ Js}) \times n = \frac{\hbar}{2\pi} n$$

c. How much energy (in eV) is stored in kinetic energy and how much in electric potential energy? Assume that the electron has zero potential energy when it is completely removed from the atom. (2 points)

$$K_n = \frac{1}{2} m v_n^2 = \frac{1}{2} (9.11 \times 10^{-31}) \frac{(2.19 \times 10^6)^2}{n^2} = \frac{1}{n^2} (2.18 \times 10^{-18} \text{ J}) = \frac{13.6 \text{ eV}}{n^2}$$

$$U_n = -\frac{ke^2}{r_n} = \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{n^2 \times (5.29 \times 10^{-11})} = \frac{(4.36 \times 10^{-8} \text{ J})}{n^2} = \frac{27.2 \text{ eV}}{n^2}$$

d. Quantum mechanics suggests that the electron is really described by a wavefunction that gives an electron probability “cloud.” One might then view the Bohr radius $r_n$ and velocity $v_n$ as measures of the uncertainty in the electron’s position and velocity.

Show that this interpretation is consistent with Heisenberg’s uncertainty relation in quantum mechanics. Which orbit comes closest to violating the constraint? (3 points)

$$\Delta p \Delta x = (mv_n)(r_n) = \frac{(9.11 \times 10^{-31})}{n} (2.19 \times 10^6)(5.29 \times 10^{-11}) n^2$$

$$= (1.05 \times 10^{-34} \text{ Js}) \times n \geq \frac{\hbar}{2\pi}$$

This will be barely true for $n = 1$, and well satisfied for all higher $n$. 
e. In the classical picture, the orbiting electron is like a current loop around the nucleus. Find the current flowing in the loop when $n = 1$. (2 points)

$$I = \frac{\Delta Q}{\Delta t} = \frac{e}{(2\pi r/v)} = \frac{e}{2\pi \frac{v}{2\pi}}$$

$$= \frac{(1.6 \times 10^{-19})(2.19 \times 10^6)}{2\pi(5.29 \times 10^{-11})} = 1.05 \times 10^{-3} \text{ A}$$

f. What is the magnetic moment of the $n = 1$ electron orbit thought of as a current loop. (1 point)

$$\mu = NIa = NI r^2 = (1)(1.05 \times 10^{-3})\pi(5.29 \times 10^{-11})^2 = 9.23 \times 10^{-24} \text{ Am}^2$$

g. Suppose now that the hydrogen atom is oriented as shown. A magnetic field is applied perpendicular to the electron orbit either (a) into the page, or (b) out of the page. Argue what will happen to the radius of the electron's orbit in each case. Which case will have the lower energy? (3 points)

(a) With the field into the page, $v \times B$ points outward, but because the electron is negative, this results in an inward force, decreasing the radius.

(b) Force opposite, increasing the radius.

(c) One might be tempted to say that the case with the smallest radius has the lowest energy, but remember that the velocity will increase to conserve angular momentum, so our simple relationships don't hold and the situation becomes complicated. However, we know that the lowest energy state will be when the magnetic moment lines up with the magnetic field. Because the negative electron is traveling clockwise, the current is going counter-clockwise and the magnetic moment is out the the page, so case (b) has a lower energy.
5. Consider the following alternating current circuit. Here \( R = 150 \, \Omega \), \( C = 1.50 \, \mu F \) and \( H = 40 \, \text{mH} \).

\[
\begin{align*}
X_C &= \frac{1}{2\pi f \, C} \\
X_L &= 2\pi f \cdot L
\end{align*}
\]

a. Write expressions for the reactances of the capacitor and the inductor in terms of \( C \) and \( L \) and the frequency \( f \) of the voltage source. (1 point)

b. Suppose the frequency is very low. Does the largest current flow in the resistor, the capacitor or the inductor? What if the frequency is very high? Why? (2 points)

\[
\begin{align*}
\text{low: inductor} \\
\text{high: capacitor}
\end{align*}
\]

c. Note that since the three components are in parallel, the same potential \( V \) is dropped across the resistor, the capacitor or the inductor. Draw a phasor diagram showing the potential \( V \) and the three currents \( I_R, I_C \) and \( I_H \). (3 points)
d. Write an expression for the total current $I$ as a function of $V$, $R$, and the reactances. (2 points)

$$I_T^2 = I_R^2 + (I_C - I_L)^2$$

$$= (\frac{V}{E})^2 + (V \cdot \omega C - \frac{V}{\omega L})^2 \Rightarrow I = V \cdot \sqrt{\left(\frac{1}{\omega L}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

e. What is the condition on the reactances which minimizes the total current? Given the values for $L$ and $C$ above, at what frequency does this occur? (2 points)

$$f = 0$$

f. Why doesn't the power dissipated in the circuit depend on the frequency $f$? If the rms voltage is 35 V, what is this power? (2 points)

On average: No power dissipates through $L \frac{1}{\omega}$. Power dissipates on $R$: $P = \frac{V^2}{R}$ (independent of frequency) $= 8.1$ W.

6. A circular loop lies in a uniform magnetic field directed into the plane of the loop. The loop is moving with a constant velocity into a region without magnetic field. Determine the direction of induced current in the loop and sketch a graph of the emf as a function of time. (4 points)

Region with uniform B field

Region with no B field

$\vec{E} = -\frac{\Delta \vec{B}}{\Delta t} = -\frac{\Delta (\vec{B} \cdot \vec{A})}{\Delta t}$

A decreases, so $\vec{E}$ causes current which reinforces $\vec{B}$. By r.h.r. this is clockwise.
7. An X-ray photon Compton-scatters off an electron that is initially at rest. The final wavelength of the photon is $6 \times 10^{-12}$ m and the final velocity of the electron is $1.2 \times 10^8$ m/s.

a. Compute the relativistic kinetic energy and momentum (amplitude only!) of the electron. (3 points)

$$K_{E} = E_{f} - E_{i} = \gamma m c^2 - m c^2 = m c^2(\gamma - 1) \quad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \gamma m v = 1.2 \times 10^{-22} \text{ kg} \cdot \text{m/s}$$

b. Find the original wavelength of the photon before the scattering. (3 points)

Conservation of energy:

$$E_{f, \text{photon}} = E_{i, \text{photon}} = K_{E} + E_{\text{kinetic}}$$

$$= \frac{h c}{\lambda} + k E_{\gamma} = 4.06 \times 10^{-17} J + 7.47 \times 10^{-17} J$$

$$\lambda_f = \frac{h c}{E_{f, \text{photon}}} = 5.90 \times 10^{-12} \text{ m}$$

8. A ray in the air is incident at an angle $\alpha = 60^\circ$ on a screen. A glass plate ($n = 1.5$) of thickness 1 cm is inserted in 5 cm in front of (and parallel to) the screen, as shown.

a. Sketch the path of the ray with the glass plate present. (2 points)

b. How far does the spot on the screen move when the glass is inserted? (4 points)

We see $\sin \theta = \frac{1}{n_1} \sin \alpha$ 

$\sin \theta = \frac{1}{1.5} \sin 60^\circ \Rightarrow \theta = 35.3^\circ$

We see $\Delta x = x_2 - x_1$

$$\tan \alpha = \frac{x_2}{t} , \quad \tan \theta = \frac{x_1}{t}$$

$$\Delta x = t (\tan \alpha - \tan \theta)$$

$$= 1 \times (1.732 - 0.473) = 1.26 \text{ cm}$$
9. a. Consider the fusion reaction

\[ ^3\text{H} + ^1\text{H} \rightarrow ^3\text{He} + ^0\text{n}. \]

What is the energy liberated in this reaction in MeV, given the masses

\[ ^0\text{n} = 1.008665 \text{u} \]
\[ ^1\text{H} = 2.014102 \text{u} \]
\[ ^3\text{He} = 3.016030 \text{u} \]

(3 points)

\[ \Delta m = 2m_{^1\text{H}} - m_{^0\text{n}} - m_{^3\text{He}} = \]
\[ = (2 \times 2.014102 - 1.008665 - 3.016030) \text{u} \]
\[ = 3.15 \times 10^{-3} \text{u} \]
\[ \Delta E = \Delta m \times \text{931.5 MeV/u} \]
\[ = 3.15 \times 10^{-3} \times 931.5 \text{ MeV} \]

b. Compute the energy in Joules liberated by the fusion of 1 kg of deuterium (\(^2\text{H}\)) into \(^3\text{He}\) via the reaction given above. (2 points)

\[ \varepsilon = \frac{1000 \times 6 \times 10^{23}}{2} \times \frac{1}{2} \times 3.15 \text{ MeV} \times 10^6 \]
\[ = 3.15 \times 10^{13} \text{J} \]

2.H fusion

c. Consider the fission reaction

\[ ^{235}\text{U} + ^{140}\text{Xe} \rightarrow ^{85}\text{Sr} + 2^0\text{n}. \]

Assuming that the binding energy per nucleon of U is about 7.6 MeV and increases to 8.5 MeV for the Xe and Sr, estimate the amount of energy released in this reaction in MeV. (3 points)

\[ \Delta E_{\text{binding}} = (8.5 - 7.6) \text{MeV} = 0.9 \text{MeV} \]

\[ \Delta E = 0.9 \text{MeV} \times 235 \approx 211 \text{ MeV} \]
d. Compute the energy in Joules liberated by the fission of 1 kg of $^{235}\text{U}$. Compare your result to what you found in part (b). (2 points)

$$E = \frac{1000 \text{ g}}{235 \text{ g}} \frac{6.2 \times 10^{-26}}{\text{g}} \frac{211 \text{ MeV}}{1 \text{ eV}} \frac{1 \text{ eV}}{1.6 \times 10^{-19}}$$

$$E \approx 9 \times 10^{18} \text{ J}$$

Comparing to (b).

e. An advanced civilization generates energy by dumping its garbage into a nearby black hole. In this process, the entire rest mass of garbage can be converted into energy with 100% efficiency. How much energy can this civilization extract from 1 kg of garbage? How does this compare with results (b) and (d)? (2.3 points)

$$E = m \gamma m_0 \gamma \frac{1}{\gamma - 1}$$

$$E = 9 \times 10^{18} \text{ J}$$

much bigger than (d).

f. How many 100 W light bulbs could the advanced civilization keep burning using 1 kg of garbage per year? (2 points)

$$t = 1 \text{ year} = \frac{365 \times 24 \times 3600 \text{ s}}{1 \text{ s}} = 3.15 \times 10^7$$

$$E = N \times 100 \text{ W} \times t$$

$$N = \frac{E}{100 \text{ W} \times t} = 2.05 \times 10^7$$
10. Consider the following combination of a concave mirror together with a converging lens. The focal length of the mirror is 4 cm. The focal length of the lens is 6 cm. The lens and the mirror are 20 cm apart.

![Diagram of mirror and lens combination]

a. For the moment, ignore the lens. Suppose an object is placed 6 cm in front of the mirror as shown. Is its image in the mirror real or virtual? Give the position of the image relative to the mirror. (2 points)

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \rightarrow \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}
\]

\[d_i = \frac{12 \text{ cm}}{12} \text{ real}
\]

inverted and left of mirror

\[
(m = d_i/d_o = 2) \quad h_o = \text{object height}
\]

\[h_i = 2 \cdot h_o
\]

b. Now consider the image of the object viewed through the lens. Two images are formed. One is the direct image of the object. The other is the image of the reflection of the object in the mirror.

What is the position, relative to the lens, of the direct image? (1 point)

Direct image

\[d_o' = 20 - 6 = 14 \text{ cm}
\]

\[
\frac{1}{d_i'} = \frac{1}{f} - \frac{1}{d_o'} = \frac{1}{6} - \frac{1}{14} = 0.095
\]

\[d_i' = 10.5 \text{ cm}
\]

Direct Image, inverted, real and 10.5 cm left of lens.

\[h_i' = \frac{10.5}{14} = 0.75 h_o
\]
c. What is the position, relative to the lens, of the image of the reflected object? (3 points)

Reflected object is \(20 - 12 = 8\) cm to right of lens. Inverted and real.

\[ d_o'' = +8 \]

\[
\frac{1}{d_i''} = \frac{1}{f} - \frac{1}{d_o''} = \frac{1}{6} - \frac{1}{8} = \frac{1}{24}
\]

\[ d_i'' = 24 \]

Final image is 24 cm to left of lens.

\[ h_i'' = \frac{24}{8} h_o'' = 3 h_o'' \]

d. Which image is larger and by what factor? (3 points)

From (a), \( h_i = 2 h_o \)

so \( h_o'' = 2 h_o \)

Reflected image size \( h_i'' = 3 h_o'' \) (see c)

\[ = 6 h_o \]

Direct image size (see (b))

\[ h_i' = 0.75 h_o \rightarrow \text{Ref. image is larger by factor } \frac{6}{0.75} = 8 \]

e. Do both images have the same orientation? Are they real or virtual? (2 points)

Reflected image is upright.

Direct image is inverted.

Both are real.