Learning Guide 1

The following formulae may be useful:

\[ F = k \frac{Q_1Q_2}{r^2} = \frac{1}{4\pi \varepsilon_0} \frac{Q_1Q_2}{r^2} \]

\[ k = 9.0 \times 10^9 \text{N m}^2\text{C}^{-2} \]

\[ \vec{F} = q\vec{E} \]

\[ \Delta P\text{E} = qV_{ba} \]

\[ V = Ed \]

\[ e = 1.6 \times 10^{-19} \text{C} \]

\[ V = kQ/r \text{ potential of point charge Q} \]

\[ Q = CV \]

\[ \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}Q^2/C \text{ energy stored in capacitor} \]

1. Alpha particles are nuclei which consist of two protons and two neutrons bound together by the nuclear force. Alpha particles are approximately spherical in shape with diameter \( \sim 2 \times 10^{-13} \text{cm} \). Assuming the protons are separated on the average by \( 1 \times 10^{-13} \text{cm} \), compute the electrostatic force of repulsion between them. Compute also the gravitational force and compare with the electrostatic force. What can you conclude about the strength of the nuclear force? (The charge and mass of a proton are \( 1.60 \times 10^{-19} \text{C} \) and \( 1.67 \times 10^{-24} \text{g} \), respectively.)

Key 28. HQ 7. HQ 5.
2. Two very small spheres, each having a mass of 0.010 g, are attached to fine silk threads 5 cm long and hung from a common point. When the spheres are given equal quantities of negative charge, each thread makes an angle of $30^\circ$ with the vertical. Compute the charge on each sphere.  

3. A thin, approximately flat membrane separates the predominantly negative cell interior from the predominantly positive cell exterior.

   (a) If the electric field in the membrane is $10^7$ N/C, find the charge per unit area in the layers on each side of the membrane. Assume the dielectric constant of the membrane is 8.  

   (b) Suppose the membrane is $10^{-8}$ m thick. Compute the capacitance per cm$^2$ of membrane area.  

   (c) If the potential difference across the membrane is 0.1 volt, compute the stored energy per cm$^2$ of area.

4. (a) If the energy for each impulse of a pacemaker, $2.4 \times 10^{-4}$ J, is stored in a capacitor charged to a potential of 6.0 V, what is the capacitance of the capacitor?  

   Key 26. HQ 12.
(b) What charge is accumulated on the capacitor before each impulse?  

(c) If 70 impulses are delivered per minute, how much energy is needed for one day of continuous operation?  

(d) What is the average power output of the pacemaker?  

5. (a) Show that the energy stored in a capacitor $C$ may be expressed as

$$ U = \frac{CV^2}{2} = \frac{Q^2}{2C}. $$

HQ 19.

(b) The parallel plate capacitor shown in Figure 1 has been charged to a potential difference $V_0$, and then is insulated from all external objects. The space between the plates is then completely filled by the insertion of a slab of paraffin of dielectric constant 6.0.

i. What is the resultant potential difference between the plates?  


ii. How does the field between the plates compare with the field before the slab was inserted?  

KEY 7. HQ 13.

(c) i. Find the energy stored in the capacitor of Figure 2 when paraffin is inserted between the plates after the voltage $V_0$ has been applied to the plates.  

KEY 33. HQ 11.

ii. Comparing this energy with the energy stored before the paraffin was inserted, determine how much work must be done by an external agent now to remove the paraffin slab.  

KEY 2. HQ 15.
Two parallel plates, each with area \( A = 100 \text{ cm}^2 \) and separated a distance \( d = 1 \text{ cm} \), are charged with \(-Q\) on one plate and \(-Q\) on the other, until the potential difference between them is \( V = 100 \text{ volts} \).

6. (a) What is the charge \( Q \)?

(b) An electron is emitted from the negatively charged plate and accelerates to the positively charged plate. What is the kinetic energy of the electron just before it hits the positive plate? (Express your energy in both joules and electron-volts.)

Key 12. HQ 10.

Helping Questions

1. What is the relation between the charge per unit area and the electric field for a parallel plate capacitor with vacuum between the plates? Key 19.

2. What is the definition of average power? Key 23.

3. What is the magnitude and direction of the electric force on one of the spheres? Key 10.

4. When the paraffin is inserted, does the charge $Q$ change? How is the charge $Q$ related to the final voltage between the plates $V_f$? Key 20.

5. What type of force is required to bind objects together? Key 22.

6. If energy is conserved, where does the change in electrical potential energy of the electron go? Key 3.

7. What is the electric force between two charges $q_1$ and $q_2$ separated by a distance $r$? Key 29.

8. What is the final velocity of the electron if you assume it starts from rest and is accelerated with the acceleration calculated in the previous helping question over a distance $d$? Key 25.

9. What is the relationship between $V_0$, $Q$, and $C$ for the capacitor in Figure 1? Solve for $Q$. Key 8.

10. What is the relation between the area of the plates, the separation of the plates, and the capacitance of the plates? Key 34.

11. What is the charge stored on the plates when $V_0$ is applied? What is the capacitance of the plate system after the paraffin is inserted? Now you should be able to get the energy stored in the capacitor. Key 13.

12. Draw a separate diagram showing the forces acting on one of the spheres and compute the tension in one of the silk threads. Key 16. HQ 3.

13. What is the value of the electric field inside a parallel plate capacitor of separation $d$ and with voltage $V_0$ between plates? How does the field change when the paraffin is inserted? Key 30.

14. What is the electric field between the plates if a dielectric of dielectric constant $K$ is added between the plates? Key 24.

15. What is the energy of the system before the paraffin is inserted? If the energy is to be conserved in the system, how much energy must be supplied by an external source to remove the paraffin block? Key 15.
16. What is the relationship between the voltage difference between the plates and the change in the electron’s potential energy as it moves the distance \( d \)? \( \text{Key 31.} \)

17. What is the force on this charge due to the electric field between the parallel plates? What is the magnitude and direction of the acceleration on the charge due to this force? \( \text{Key 17.} \)

18. Express the energy stored in a capacitor in terms of the capacitance \( C \) and the potential difference across the plates \( V \). \( \text{Key 21.} \)

19. What is the relationship between the charge \( Q \) on the capacitor, voltage \( V \) across the capacitor, and its capacitance \( C \)? How does the energy stored in a capacitor depend on the charge stored \( Q \) and the voltage \( V_0 \)? \( \text{Key 1.} \)
Solutions

1. $C = Q/V, U = QV/2$.
2. $W = \Delta U = 0.42CV_0^2$.
3. Change in potential energy equals change in kinetic energy.
4. $Q = CV = 8.0 \times 10^{-5}$ C.
5. $C = 2E/V^2 = 13\mu$F.
6. $P = (E/min)(\# \text{ sec/min}) = 2.8 \times 10^{-4}$ watt.
7. $E_0 = V_0/d, E_f = V_f/d$, so $E_f/E_0 = V_f/V_0 = 1/6$.
8. $V_0 = Q/C, Q = V_0C$.
9. $C/A = K\varepsilon_0/l = 0.7\mu$F cm$^{-2}$.
10. $F = \frac{kq^2}{(2l \sin 30^\circ)^2}$ = ...
11. $Q/A = K\varepsilon_0E_{eff} = 7.08 \times 10^{-4}$ C m$^{-2}$.
12. $Q = VC = V \left( \frac{K\varepsilon_0A}{d} \right) = 100\varepsilon_0100 \times 10^{-4}$m$^2$
    $\quad = 8.84 \times 10^{-10}$ C.
13. $Q = V_0C; C_{after} = K C_{before}$.
14. $U/A = CV^2/2A = 3.5 \times 10^{-9}$ J cm$^{-2}$.
15. $U(\text{without paraffin}) = V_0^2C/2, U(\text{with paraffin}) = V_0^2C/2K$,
    $\Delta U(\text{external}) = \frac{V_0^2C}{2} \left( 1 - \frac{1}{K} \right)$.
16. $T = mg / \cos \theta$

17. Force = $q\vec{E} = q\Delta v / d$ in $x$ direction; $\vec{a} = \vec{F} / M = q\Delta v / Md$ in $+x$ direction.

18. K.E. = $W = 1.6 \times 10^{-17}$ joules = 100 eV.

19. \[ Q/A = \varepsilon_0 E. \]

20. Charge does not change. \[ V_f = Q/C_f = Q/KC. \]

21. \[ U = CV^2 / 2. \]

22. Attractive.

23. \[ P = \Delta E / \Delta t. \]

24. \[ \vec{E}_{\text{eff}} = \vec{E} / K. \]

25. \[ v_f^2 = v_0^2 + 2ad = 2qV / M. \]

26. \[
    Q = \left( \frac{(2l \sin 30^\circ)^2 mg \sin 30^\circ}{k \cos 30^\circ} \right)^{1/2} = 4.0 \times 10^{-9} \text{ C}
\]

27. \[ V_f = Q_f / C_f = Q_0 / 6 C_0 = V_0 / 6. \]

28. $F_{\text{electric}} = kQ_1^2 / r^2 = 230 \text{ N}; F_{\text{grav}} = GM^2 / r^2 = 1.9 \times 10^{-34} \text{ N}; F_{\text{grav}} \ll F_{\text{electric}}$. The nuclear force must be attractive and much stronger than 230 N.

29. \[ F = kq_1 q_2 / r^2 \]

30. \[ E_0 = \Delta V / d = V_0 / d, \quad E_f = V_f / d = V_0 / Kd = E_0 / K. \]

31. The change of energy is $q\Delta v$.

32. \[ E = 24 \text{ J}. \]

33. \[ U = CV_0^2 / 2K = CV_0^2 / 12. \]

34. \[ C = K\varepsilon_0 A / l. \]