Quantum Mechanics

Physics 102
18 April 2002
Lecture 9

Planck
Bohr
Schroedinger
Heisenberg

From: http://www.th.physik.uni-frankfurt.de/~jr/portraits.html
Blackbody radiation

Recall: a perfect blackbody absorbs and re-emits all radiation that falls upon it.

All blackbodies at a given temperature emit the same power in the same frequency band, i.e. they all have the same characteristic spectrum.

The frequency at which most power is emitted is proportional to the temperature.

The total power emitted increases like $T^4$. As you may recall from last semester: $Q = \sigma_B T^4 At$

At the end of the 1800s, no one could predict the radiated power, $Q$, as a function of frequency.
Blackbody radiation cont.

In 1900, Planck found that by assuming the blackbody was made up of tiny atomic oscillators that could absorb or emit discrete amounts of energy with \( E = hf \) he could match the observed spectrum. Energy is **quantized** as *photons* in units of \( E = hf \)

\[
I_f = \frac{2hf^3}{c^2} \frac{1}{e^{\frac{hf}{k_BT}} - 1}
\]

with: \( h = 6.6 \times 10^{-34} \text{Js} \)

Spectrum of the oldest light in the universe!

FYI:
The hot filament of an incandescent bulb is a good source of blackbody radiation. When observed through a diffraction grating, a broad spectrum is seen. When the temperature of the filament is decreased:

A. The intensity of the spectrum decreases but the relative intensities at all frequencies (colors) stay the same.

B. The intensity of the spectrum remains the same at high frequencies (blue) but there is less red light.

C. The intensity decreases at all frequencies though blue light is diminished more than red light.

D. The intensity decreases at all frequencies though red light diminishes more than blue.
The reality of photons: photoelectric effect

Light (electromagnetic radiation) impinging on a metal ejects electrons from its surface.

Ejection of electrons depends on light’s frequency, with a minimum frequency required to eject any. If the frequency is too low, no electron is ejected regardless of intensity.
Light is passed through a quartz prism. The energy of the dispersed light is measured in five locations 1-5. Which of the following is true:

A. There is only energy where we see light in locations 2, 3, & 4.

B. The energy of the photons is higher in the low number locations (UV) than in the high number locations (infrared)

C. We use quartz to enhance the dispersion and transmission of visible light and consequently give more energy to all the photons.

D. The energy of the photons increases in going from the infrared (location 1) to the UV (location 5).
Observe that there is current only when UV shines on the cathode: photons need a minimum energy to photoeject electrons.
Photoelectric effect cont.

The *number* of electrons ejected depends on *intensity*, but the energy of the ejected electrons is independent of energy.

The wave model cannot explain the photoelectric effect. Einstein explained it using *photons* with \( E=hf \).

- A photon ejects an electron if it has the energy.
- Intensity \( \propto \) number of *photons* \( \propto \) number of electrons.

For ejected electrons, \( KE=hf-W_0 \) where \( W_0 \) is the work function and depends on the metal.
The momentum of a photon

Arthur Compton (Ph.D. Princeton 1916) demonstrated that photons have **momentum** as well as **energy**.

- From relativity, we expect that \( p = E/c \) for photons
- Imagine a photon colliding with an electron at rest
  - conservation of energy and
  - conservation of momentum
  - imply (using relativistic formulae for electron):
    \[
    \lambda' - \lambda = \frac{h}{mc} (1 - \cos \theta)
    \]
    (recall that \( \lambda f = c \))

longer wavelength: lower energy
A very weak light source emits a single photon in the direction of a Young double-slit apparatus. The film behind the double slit will register:

A. A single dot behind one or the other slit
B. An interference pattern produced by the double slit.
C. Neither of the above.

There seems no question that light acts as though it is made of particles—photons—that carry momentum and energy.
Wave-Particle Duality

So is light a wave or a particle?

- In some circumstances it behaves like a particle (photoelectric effect, Compton effect, …)
- In others it behaves like a wave (Young double slit, diffraction, …)

“I think I can safely say that nobody understands quantum mechanics” - Richard Feynman (Princeton Ph.D. 1942)
The Wave Nature of Matter

- By analogy, de Broglie (1924) suggested that matter may have wave-like properties, with $\lambda = \frac{h}{p}$.
- Davisson (Ph.D. Princeton) and Germer demonstrated diffraction of electrons by crystals in 1925.
- Electrons passing through a double slit act like light.

Tonomura et al AJP 57, 1989
A beam of electrons is shot toward a crystal with an interatomic spacing of 0.1 nm. Diffraction spots are seen with a regular spacing of 0.1 radians. The electrons have a wavelength of about:

A. 100 nanometer  
B. 1 nanometer  
C. 0.01 nanometer  
D. 0.0001 nanometers

From which we can derive their momentum through

\[ p = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{\lambda} \text{ Js} \]
Particles act like waves and obey the laws of diffraction.

Light acts like a particle with zero rest mass and collides with massive particles much as another massive particle would.

To understand the basis of our reality we have to ascribe to the fundamental constituents of Nature properties of which we have no direct experience.

At the quantum level, Nature is fundamentally different than at the “macro” level.
The Heisenberg Uncertainty Principle

- The narrower the slit we try to shine light through, the wider the diffraction pattern that results: $\theta \propto \lambda / w$
- The same is true for particles (since they are waves!): constraining the position increases the scatter in the momentum.

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

- The uncertainty principle puts a limit on how well two “complementary” quantities can be measured simultaneously.
Atomic Structure: The Nucleus

The “plum-pudding” model (ca. 1900) imagined electrons embedded in a jelly-like positive medium.

Rutherford, Geiger, and Marsden found that alpha particles fired at a thin gold foil occasionally scattered backwards.

This implied a dense, heavy nucleus.
The Bohr Atom

- Bohr assumed electrons orbited nucleus in circular paths and that the only stable orbits had angular momentum in integer units of

\[ L_n = m_e v_n r_n = n \frac{h}{2\pi} \]

- Using Coulomb’s law, you should show (and know!) that

\[ E_n \propto \frac{1}{n^2} \]

- An atom emits or absorbs a photon of energy \( E_{n1} - E_{n2} \) when it jumps from one stable orbit to another (discontinuously).
Line Spectra and Electron Orbits

- An atom emits or absorbs quanta of energy when an electron jumps from one allowed energy level to another.
- The quanta show up at discrete frequencies, \( f = \frac{\Delta E}{h} \) characteristic of the type of atom.

![Diagram of energy levels and transitions in hydrogen](image)

You can see this!
De Broglie Waves and the atom

- Using de Broglie’s wavelength for an electron \( \lambda = \frac{h}{p} \)
  and the quantization of angular momentum \( L = \frac{nh}{2\pi} \)
  we find:
  \[
  L = pr = \frac{h}{\lambda} r = \frac{nh}{2\pi}
  \]
  Thus: \( n\lambda = 2\pi r \)

- This has a natural graphical interpretation:
  allowed orbits are have circumference equal to an integer number of wavelengths

- Think *standing waves*!
Quantum Mechanics

- Particles are described by “wave functions” $\psi$ that have a complex value at each point of space.
- The magnitude of the wave function squared describes the probability of detecting the particle at a given spot.
- The wave function evolves deterministically according to Schrödinger’s equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + PE \cdot \psi$$

- Solving this equation leads to a determination of the possible orbits of the electrons and in turn to the “quantum numbers” in sec 30.5. These quantum numbers allow us to classify atoms and their bonds.
After quantum

- P.A.M. Dirac unified relativity with quantum mechanics and gave birth to “field theory”
- Field theory was used to describe a wide variety subatomic phenomena. In fact, the 1999 Nobel prize was for work in field theory.
- In theory today, the hot topic is “string theory” in which the elemental building blocks of Nature are tiny vibrating strings that live in a multidimensional (e.g. 10) dimensional space. The extra dimensions are “curled up” so we can’t experience them.