PHYSICS DEPARTMENT, PRINCETON UNIVERSITY

PHYSICS 505 MIDTERM EXAMINATION

October 25, 2012, 11:00am–12:20pm, Jadwin Hall A06

This exam contains two problems. Work both problems. They count equally although they may not be the same difficulty.

Do all the work you want graded in the separate exam books.

The exam is closed everything: no books, no notes, no calculators, no computers, no cell phones, no ipods, etc.

Write legibly. If I can't read it, it doesn't count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

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1. **Double Delta Functions.** A particle of mass m, moving in one dimension, is confined by a double delta function potential,

$$V(x) = -\frac{\hbar^2 b}{m}\left(\delta(x+a) + \delta(x-a)\right) ,$$

where $b > 0$. The energy of the particle is $E < 0$ with $-E = (\hbar^2\kappa^2/2m)$.

(a) Since the potential has even parity, bound states have even or odd parity. What are the general even parity ($\psi(x) = +\psi(-x)$) and odd parity ($\psi(x) = -\psi(-x)$) bound state wave functions? (Don't worry about normalization.) What are the matching conditions at $x \pm a$ for wave functions of each parity? (Note, you will solve the conditions later, here the question is "what are the conditions?")

(b) For even parity wave functions, show that there is exactly one bound state and obtain an equation whose solution determines κ in terms of b and a.

(c) For odd parity wave functions, what is the condition on b and a such that a bound state exists? Obtain an equation whose solution determines κ when the bound state exists.

2. **Well!** Consider an infinitely deep (one-dimensional) potential well, $V(x) = 0$ for $0 \le x \le 2a$ and $V(x) = \infty$ for $x < 0$ or $x > 2a$. A particle of mass m moves in this potential well.

(a) The energy eigenvalues and time independent eigenfunctions can be labeled by an integer $n, n = 1, 2, 3, \ldots$, where the energy increases as n increases. What are the eigenvalues, E_n, and the corresponding (normalized) eigenfunctions, $\psi_n(x)$? What is the interpretation of n?

(b) At $t = 0$, the wave function of the particle in the well is

$$\psi(x, t = 0) = \sqrt{\frac{2}{a}}\sin\frac{\pi x}{a} , \text{ for } 0 \le x \le a , \quad \text{and } \psi(x, t = 0) = 0 , \text{ otherwise.}$$

That is, the particle is in the left half of the well. (This state might arise, for example, if the particle were in the ground state of a well of width a and the width of the well were suddenly doubled to $2a$ by moving the right boundary.) What is the wave function for $t > 0$? Also, if a measurement of the energy of the particle is made after $t = 0$, what is the probability of obtaining E_n, where n is one of the energy eigenvalues you obtained in part (a)? Hint: you really should know your trig identities, but in case you don't remember,

$$\sin A \sin B = (\cos(A - B) - \cos(A + B))/2 .$$

(c) The probability density oscillates with period T. What is T? Hint: you can solve this part without having done part (b).

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