PHYSICS DEPARTMENT, PRINCETON UNIVERSITY

PHYSICS 505 MIDTERM EXAMINATION

October 27, 2011, 11:00am–12:20pm, Jadwin Hall A06

This exam contains two problems. Work both problems. They count equally although they may not be the same difficulty.

Do all the work you want graded in the separate exam books.

The exam is closed everything: no books, no notes, no calculators, no computers, no cell phones, no ipods, etc.

Write legibly. If I can't read it, it doesn't count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

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1. Particles of mass m move in one dimension (the x direction) with energy E and wave number $\pm k$ with $E = \hbar^2 k^2/2m$. They move under the influence of a potential which consists of δ-functions at $x = \pm a$.

$$V(x) = \frac{\hbar^2 b}{m}\delta(x+a) + \frac{\hbar^2 b}{m}\delta(x-a)\,.$$

The wave function for $x < -a$ can be written as $Ae^{+ikx} + Be^{-ikx}$ and the wave function for $x > +a$ is $Ce^{+ikx} + De^{-ikx}$.

(a) The relation between the coefficients A and B and the coefficients C and D can be written in matrix form as

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix}\,.$$

An incident wave comes from $x = -\infty$. In terms of the matrix components above, what are the reflection and transmission coefficients?

(b) Determine the matrix introduced in part (a).

(c) In terms of k, a, and b, what is the condition for 100% transmission of the incident wave. You may leave your answer as a transcendental equation, but it should be simplified as much as possible.

2. A one dimensional particle is confined to a box between $x = -a$ and $x = +a$ by the potential $V(x) = 0$ for $|x| < a$ and $V = \infty$ for $|x| > a$. The mass of the particle is m. The wave function at $t = 0$ is 0 for $|x| > a$ (of course!) and for $|x| < a$ it is

$$\psi(x,0) = \frac{1}{\sqrt{6a}}\left(\sqrt{3}\cos\frac{\pi x}{2a} + \sqrt{2}\sin\frac{\pi x}{a} + \cos\frac{3\pi x}{2a}\right)\,.$$

(a) What is the wave function at time t?

(b) A measurement of the energy is made. What are the possible values of the energy that can result when the wave function is ψ? What is the probability of obtaining each energy?

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