1. 1D ionization. This problem is from the January, 2007, prelims. Consider a non-relativistic mass m particle with coordinate x in one dimension that is subject to an attractive delta-function potential at $x = 0$, *i.e.*, a potential $V(x) = -V_0\delta(x/a)$, with $V_0 > 0$.

(a) The ground state of the particle is a bound state. Find its wave function and binding energy.

(b) The particle is now perturbed by a weak time-dependent potential $V(x,t) = Fx\cos\omega t$. Find the transition rate from the bound state to the continuum. (It may help to confine the particle in a large box $|x| < L/2$ and take the limit $L \to \infty$.)

2. Spherical Square Potential. Consider low energy scattering of a particle of mass m from a spherical potential of radius a:

$$V(r) = \begin{cases} V_0 & r < a \\ 0 & r > a \end{cases},$$

where V_0 may be either positive or negative.

(a) Calculate the s-wave phase shift for incident energy E. Note that low energy scattering means $ka \ll 1$.

(b) Can the s-wave phase shift be a multiple of π? What happens in this case? Hint: Google "Ramsauer Effect."

3. A really, really square potential! This problem appeared on the May, 2004 prelims. A beam of particles of mass m and energy E propagates along the z axis of a coordinate system and scatters from the cubic potential

$$V = \begin{cases} v & \text{if } |x| \le L,\ |y| \le L, \text{ and } |z| \le L, \\ 0 & \text{otherwise} \end{cases}$$

where v is a small constant energy.

(a) Use the Born approximation to find an explicit formula for the scattering cross section $\sigma = \sigma(\theta,\phi)$ as a function of the angles θ and ϕ. Recall that spherical coordinates of a point in space are related to the Cartesian coordinates (x, y, z) by $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, and $x = r\cos\theta$. The Born approximation is easy to evaluate in one coordinate system and hard in the other.

(b) Under what circumstances is this approximation for the scattering cross section valid? Explain.

4. Neutron capture. (Based on a problem in Dicke and Witke, *Introduction to Quantum Mechanics*.) For a particular nucleus, the neutron absorption cross section, for 0.1 eV neutrons, is $\sigma_a = 2.5 \times 10^{-18}\,\mathrm{cm}^2$. What are the upper and lower bounds on the 0.1 eV neutron elastic scattering cross section?