1. This problem is from the May, 2004 Prelims. Hydrogen molecule. Consider the neutral hydrogen molecule, H_2. Write down the Hamiltonian keeping only the kinetic energy terms and the Coulomb interactions of all the constituents and omitting terms which cause fine and hyperfine structure.

(a) What is the degeneracy of the ground state? Give all quantum numbers and symmetries of the ground state(s), including the electron and proton spin degrees of freedom.

(b) What is the degeneracy and what are all the quantum numbers of the first excited state of this H_2 molecule? Explain.

(c) What is the energy difference between ground and first excited states? Estimate it first through a formula, in terms of properties of the molecule's ground state, and then in electron-Volts (eV).

2. With Hund's rules, determine S, L, and J for nitrogen whose configuration is $1s^2\,2s^2\,2p^3$.

3. Two particles of mass m are confined to a rectangular box of sides $a < b < c$. They are in the lowest energy state compatible with the conditions in the cases below. For each of these cases, determine the lowest energy state and its energy and also use first order perturbation theory to determine the correction to the energy if there is an interaction between the particles of the form $V = (V_0(abc))\delta^{(3)}(\boldsymbol{r}_1 - \boldsymbol{r}_2)$.

(a) Two non-identical particles.

(b) Two identical particles of spin 0.

(c) Two identical particles of spin 1/2 in the singlet state.

(d) Two identical particles of spin 1/2 in the triplet state.

4. Beta decay rearrangement. Based on a problem in Schwabl. In β-decay, the nuclear charge number Z of a $Z - 1$ times ionized atom (so it's hydrogen-like!) changes suddenly to $Z + 1$. Calculate the probabilities for the transition of the electron to the $2s$ or $3s$ states given that the electron was in the ground state before the β-decay. Do you have any comments on energy conservation?

5. Coulomb excitation. Somewhat based on a problem from Schwabl. Consider hydrogen in its ground state at $t = -\infty$. It's acted on by an electric field in the z-direction of the form

$$\boldsymbol{E}(t) = \frac{E_0 \boldsymbol{e}_z}{1 + t^2/\tau^2} \,.$$

This field can be represented by the potential $\phi = -E(t)z$. This is an approximation to what happens when a charge particle passes nearby. If it's not relativistic, we can ignore its magnetic field. What is the probability that the electron winds up in the $2p$ state at $t = +\infty$? Once you've produced a formula, make some reasonable assumptions and calculate a numerical estimate of the probability to see if a Coulomb collision is a reasonable way to excite an atom.