PHYSICS DEPARTMENT, PRINCETON UNIVERSITY

PHYSICS 505 FINAL EXAMINATION

January 18, 2012, 1:30–4:30pm, A06 Jadwin Hall

This exam contains five problems. Work any three of the five problems. All problems count equally although some are harder than others. Do all the work you want graded in the separate exam books. *Indicate clearly which three problems you have worked and want graded.* I will only grade three problems. If you hand in more than three problems without indicating which three are to be graded, I will grade the first three, only!

The exam is closed everything: no books, no notes, no calculators, no computers, no cell phones, no ipods, etc.

Write legibly. If I can't read it, it doesn't count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

If you finish early, do not leave your exam books in the room. Instead, take them to Jessica Heslin in room 210 Jadwin Hall.

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1. Quantum Zeno. In this problem we explore the "Quantum Zeno Effect." Zeno's arrow paradox is basically that when one observes an arrow in flight, it is at a particular spot at a particular instant and hence can't be moving. Not sure about Zeno, but in quantum mechanics, sufficiently rapid observations of a system can keep it from changing its state!

An electron spin interacts with a constant magnetic field. The Hamiltonian is

$$H = \mu_B \boldsymbol{B} \cdot \boldsymbol{\sigma} ,$$

where the g-factor of the electron has been taken to be 2, μ_B is the Bohr magneton, and $\boldsymbol{\sigma}$ is the vector of Pauli spin matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} .$$

The magnetic field is in the x-direction and has magnitude B, so the Hamiltonian becomes

$$H = \hbar\omega \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ,$$

where $\hbar\omega = \mu_B B$. $|{\uparrow}\rangle$ is the state with spin pointing in the $+z$-direction and $|{\downarrow}\rangle$ is the state with spin pointing in the $-z$-direction.

This problem is concerned with idealized measurements which determine the state of the spin along the z-axis. The measurements take a very short time to perform and if the state is found to be $|{\uparrow}\rangle$, the electron is left in $|{\uparrow}\rangle$ immediately after the measurement. Similarly, if the state is found to be $|{\downarrow}\rangle$, the electron is left in $|{\downarrow}\rangle$ immediately after the measurement.

The electron is prepared in state $|{\uparrow}\rangle$ at $t = 0$.

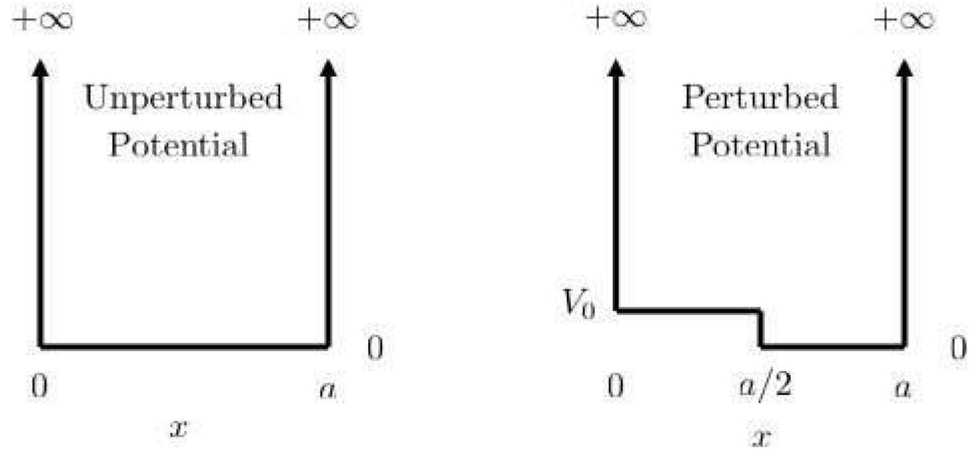
(a) What is the probability of finding the electron spin to have flipped to the state $|{\downarrow}\rangle$ at time $t_1 > 0$?

(b) The spin, starting from whatever state the measurement at t_1 left it in (either $|{\uparrow}\rangle$ or $|{\downarrow}\rangle$), continues to evolve in the magnetic field to time $t_2 = t_1 + \tau$. At t_2, the z-component of the spin is measured again. What is the probability of finding the electron in state $|{\downarrow}\rangle$ for this measurement?

(c) Starting again from $t = 0$ in state $|{\uparrow}\rangle$, measurements are made repeatedly at intervals of $\tau \ll 1/\omega$ up to time $T \gg \tau$ and with $T\omega \approx 1$. That is, measurements are made at $t_n = n\tau$ with $n = 1, 2, 3, \ldots, T/\tau$. Estimate τ such that the probability of being in the state $|{\downarrow}\rangle$ at $t = T$ is less than $p = 0.01$. Hint: the key word here is *estimate*.

2. Perturbed boxes. A particle of mass m moves in one dimension and is confined to a "box" by the potential $V = 0$ for $0 < x < a$ and $V = \infty$ for $x < 0$ or $x > a$.

(a) Prior to $t = 0$, the particle is in its ground state with energy E_1. From $t = 0$ to $t = T$, a perturbation is applied: for $0 < x < a/2$, the potential is changed from 0 to V_0 with $V_0 \ll E_2 - E_1$, where E_2 is the energy of the first excited state of the particle in the unperturbed potential well. After $t = T$, the potential is the same as it was before



$t = 0$. Find the probability that for $t > T$ the particle is in the first excited state (of the unperturbed potential) to the lowest non-vanishing order in $V_0/(E_2 - E_1)$. Hint: rather than blindly applying the golden rule, work this out from perturbation theory and Schroedinger's equation!

(b) Suppose instead, that a perturbation of the same shape as in part (a) is applied, but its strength is increased very slowly from zero up to an amplitude V_s much, much larger than $E_2 - E_1$. Upon reaching V_s, the perturbation is suddenly removed. In this case, what is the probability that the particle (which again was in the ground state before the perturbation was applied) winds up in the first excited state. Hint: the fact that the perturbation was applied slowly allows you to know what the state is at the instant before the perturbation is removed. Why?

3. Coupled angular momentum states. A two particle system is in a state $|\psi_0\rangle$, where each particle has orbital angular momentum quantum numbers $l = 1$ and $m_l = 0$. The two particles are **not** identical. The total angular momentum is denoted by $\boldsymbol{J} = \boldsymbol{L}_1 + \boldsymbol{L}_2$ and the eigenvalues of J^2 are $\hbar^2 j(j+1)$.

(a) The two particle state may be expanded in eigenstates of J^2. What values of j have non-zero amplitude in the expansion? For each of these values, what is the probability that it will be found in a measurement of J^2?

At $t = 0$, a coupling between the angular momenta is turned on so the system has Hamiltonian,

$$H = \gamma \boldsymbol{L}_1 \cdot \boldsymbol{L}_2 \,,$$

where γ is a constant. The state of the system now depends on time, $|\psi(t)\rangle$.

(b) Show that $|\langle\psi(t) \mid \psi_0\rangle|^2$ is periodic with period T and find T. Also evaluate $|\langle\psi(t) \mid \psi_0\rangle|^2$ at $t = T/2$

4. Coulomb excitation. Consider hydrogen in its ground state at $t = -\infty$. It's acted on by an electric field in the z-direction of the form

$$\boldsymbol{E}(t) = \frac{E_0 \boldsymbol{e}_z}{1 + t^2/\tau^2} \,.$$

This field can be represented by the potential $\phi = -E(t)z$. This is an approximation to what happens when a charge particle passes nearby. If it's not relativistic, we can ignore its magnetic field. What is the probability that the electron winds up in the $2p$ state at $t = +\infty$?

Useful data: for hydrogen,

$$R_{10}(r) = 2\left(\frac{1}{a}\right)^{3/2} e^{-r/a} \,,$$

$$R_{20}(r) = 2\left(\frac{1}{2a}\right)^{3/2}\left(1 - \frac{r}{2a}\right) e^{-r/2a} \,,$$

$$R_{21}(r) = \frac{1}{\sqrt{3}}\left(\frac{1}{2a}\right)^{3/2} \frac{r}{a} e^{-r/2a} \,,$$

$$a = \frac{e^2}{\hbar c} \,.$$

Also,

$$\int_{-\infty}^{+\infty} \frac{e^{-i\alpha z}}{1 + z^2}\, dz = \pi e^{-|\alpha|} \,.$$

5. Scattering from a spherical shell. A particle of mass m and energy $E = \hbar^2 k^2/2m$ is scattered by the fixed, spherically symmetric potential

$$V(r) = -V_0 a \delta(r - a) ,$$

where V_0 and a are a positive constants. In the following, use suitable approximations.

(a) What is the total scattering cross section at very low energies ($ka \ll 1$)?

(b) What is the differential cross section at very high energies ($ka \gg 1$)?

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