PHYSICS DEPARTMENT, PRINCETON UNIVERSITY

PHYSICS 505 FINAL EXAMINATION

January 18, 2013, 1:30–4:30pm, A06 Jadwin Hall

This exam contains five problems. Work any three of the five problems. All problems count equally although some are harder than others. Do all the work you want graded in the separate exam books. *Indicate clearly which three problems you have worked and want graded.* I will only grade three problems. If you hand in more than three problems without indicating which three are to be graded, I will grade the first three, only!

The exam is closed everything: no books, no notes, no calculators, no computers, no cell phones, no ipods, etc.

Write legibly. If I can't read it, it doesn't count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

If you finish early, do not leave your exam books in the room. Instead, take them to Ed Groth in room 357 Jadwin Hall.

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1. Crossed $\boldsymbol{E}$ and $\boldsymbol{B}$ fields. Consider a particle with mass m and charge e moving in uniform $\boldsymbol{B} = B\boldsymbol{e}_z$ and $\boldsymbol{E} = E\boldsymbol{e}_x$ fields with $E < B$. Use the gauge $\boldsymbol{A} = Bx\boldsymbol{e}_y$. The Hamiltonian is

$$H = \frac{1}{2m}\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right)^2 - eEx \, .$$

Find the eigenfunctions and eigenvalues for this Hamiltonian. Note that if you should find that the eigenfunctions involve standard functions which we already know about, you don't have to write out each one explicitly. For example, if you should find (you won't) that the eigenfunctions involve the Legendre polynomials $P_l(y)$, you can just leave $P_l(y)$ in your result rather than explicitly write out what $P_l(y)$ is. Of course, you have to say that P_l stands for a Legendre polynomial of order l.

Hint: you might start by reminding yourself what the classical motion looks like.

2. Spins in Positronium. Positronium is a bound state of an electron and its positively charged antiparticle, the positron. The antiparticle has the same spin and mass as the electron, just the opposite charge. This problem concerns the spins of the electron and positron when positronium is in its spatial ground state in a magnetic field in the z-direction. The Hamiltonian for the spins can be written as

$$H_s = \alpha \frac{4}{\hbar^2} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + \beta \frac{2}{\hbar}\left(S_{1z} - S_{2z}\right) \, ,$$

where α and β are constants. The first term is analogous to the hyperfine interaction in ordinary hydrogen and the second term represents the interaction with the magnetic field. $\boldsymbol{S}_1$ is the spin operator for one of the particles, say the the electron, and $\boldsymbol{S}_2$ is the spin operator for the other particle. Each spin may be up or down along the z-axis, so there are a total of four possible states: $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, and $|\downarrow\downarrow\rangle$, where the first slot represents the spin of particle 1, the electron, and the second slot represents the spin of particle 2, the positron. These are not necessarily the eigenstates of H_s.

(a) To start with, suppose the magnetic field is zero, $\beta = 0$. What are the eigenvalues and eigenstates (as combinations of the four states listed above) of H_s for this case?

(b) Now suppose the magnetic field is non-zero, $\beta \neq 0$. What are the eigenvalues of H_s for this case? The states you found in part (a) might be a good starting basis. Some of the eigenstates are messy to calculate and some are easy. So, instead of determining the eigenstates, discuss what happens to the states in the presence of a magnetic field, $\beta \neq 0$.

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3. Electric pulse. Consider hydrogen in its ground state for $t \leq 0$. It's acted on by an electric field pulse in the x-direction of the form

$$\boldsymbol{E}(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ E_0 \boldsymbol{e}_x & \text{if } 0 < t \leq \tau \\ 0 & \text{if } \tau < t \end{cases} ,$$

where E_0 and τ are positive constants. Note that this field points in the x direction, not the z direction! Such a field can be produced by placing the atom between the plates of a capacitor and pulsing the capacitor. Obtain first order expressions for the probability that the atom winds up in the 2s state and the probabilities that the atom winds up in each of the three 2p states, $|n, \ell, m\rangle = |2,1,1\rangle$, $|2,1,0\rangle$, $|2,1,-1\rangle$ for $t > \tau$.

Useful data: for hydrogen,

$$R_{10}(r) = 2\left(\frac{1}{a}\right)^{3/2} e^{-r/a} \qquad Y_{00} = +\sqrt{\frac{1}{4\pi}} ,$$

$$R_{20}(r) = 2\left(\frac{1}{2a}\right)^{3/2}\left(1 - \frac{r}{2a}\right) e^{-r/2a} \qquad Y_{10} = +\sqrt{\frac{3}{4\pi}} \cos\theta ,$$

$$R_{21}(r) = \frac{1}{\sqrt{3}}\left(\frac{1}{2a}\right)^{3/2} \frac{r}{a} e^{-r/2a} \qquad Y_{1\pm1} = \mp\sqrt{\frac{3}{8\pi}} \sin\theta \, e^{\pm i\phi} ,$$

$$a = \frac{\hbar^2}{me^2} .$$

4. Tritium Decay. Tritium is the "extra-heavy" isotope of hydrogen with a nucleus containing one proton and two neutrons. The nucleus is unstable and decays to ^{3}He by emitting an electron and an electron type anti-neutrino. (FYI, the half life of this decay is about 12.3 years and the energy release is about 18.6 keV.) Suppose a tritium atom is in its atomic ground state when the nuclear decay occurs. Assume that the decay is sufficiently rapid that the wave function of the atomic electron is unchanged by the decay and that the change of mass and the recoil of the nucleus are can be neglected. In other words, the only effect on the atomic electron is the sudden change in the charge of the nucleus, from $+e$ to $+2e$.

(a) With an electron moving in the Coulomb potential, the ground state wave function is $\psi_0(r, \theta, \phi) = Ae^{-\kappa r}$. Determine A and κ for hydrogen and for singly ionized helium (the result of the decay).

(b) What is the probability that immediately after the decay of the nucleus, the atomic electron is in the ionized helium ground state?

(c) What is the expectation value of the energy of the atomic electron in the ionized helium immediately after the decay?

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5. A non-relativistic particle of mass m scatters from a spherical square well given by the potential $V(r) = -V_0 < 0$ for $r < a$ and $V(r) = 0$ for $r > a$. We are considering low energy scattering and you may take the limit $\lambda/a \to \infty$ at the appropriate point in your calculation. λ is the de Broglie wavelength of the particle.

(a) In this limit obtain the differential cross section $d\sigma/d\Omega$ and the total cross section σ.

(b) This limiting zero-energy cross section diverges to ∞ for certain values of V_0. What are these values of V_0 and what is the physical significance of such divergences?

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