PHYSICS DEPARTMENT, PRINCETON UNIVERSITY

PHYSICS 505 MIDTERM EXAMINATION

October 22, 2009, 11:00am–12:20pm, Jadwin A06

This exam contains two problems. Work both problems. They count equally although they may not be the same difficulty.

Do all the work you want graded in the separate exam books.

The exam is closed everything: no books, no notes, no calculators, no computers, no cell phones, no ipods, etc.

Write legibly. If I can't read it, it doesn't count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

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1. Well! Consider an infinitely deep (one-dimensional) potential well, $V(x) = 0$ for $-a \le x \le +a$ and $V(x) = \infty$ for $x > |a|$. A particle of mass m moves in this potential well.

(a) The energy eigenvalues and time independent eigenfunctions can be labeled by an integer $n, n = 1, 2, 3, \ldots$, where the energy increases as n increases. What are the eigenvalues, E_n, and the corresponding (normalized) eigenfunctions, $\psi_n(x)$? What is the interpretation of n?

(b) At $t = 0$, the system is prepared in the superposition state $\psi(x, 0) = (\psi_1(x) + \psi_2(x))/\sqrt{2}$, where ψ_1 and ψ_2 are the wave functions of the ground state and first excited state you found in part (a). What is $\psi(x, t)$ and what is the time dependence of the expectation value of x, $\langle x(t) \rangle$, when the system is in state $\psi(x, t)$? (The trig identity

$$\cos A \sin B = (\sin(A + B) - \sin(A - B))/2 \,,$$

will be useful.)

(c) The square well is "enhanced" by the addition of a δ-function potential at the center of the well, so the potential is $V(x) = \lambda\delta(x)$ for $-a \le x \le +a$ and $V(x) = \infty$ for $|x| > a$, with $\lambda \ge 0$. Some of the eigenvalues and eigenfunctions you found in part (a) are unaffected by this addition to the potential. Others are changed. Which are unaffected and why? For those that are changed, find a transcendental equation whose solution would allow you to determine the new energy eigenvalues. By considering the behavior of this equation as a function of λ, show that, although the altered potential changes the energy eigenvalues, it does not change the ordering of the energy eigenvalues.

2. "Top hat" momentum distribution. A free particle of mass m moves in one dimension. The distribution of wave numbers is uniform between $k_0 - q$ and $k_0 + q$ and 0 for $|k - k_0| > q$. That is, the wave function in wave number space is

$$\varphi(k) = \begin{cases} 1/\sqrt{2q} \,, & |k - k_0| \le q; \\ 0 \,, & |k - k_0| > q. \end{cases}$$

(a) What is the uncertainty in the momentum?

(b) What is the configuration space wave function, $\psi(x)$, at $t = 0$?

(c) The uncertainty in position (at $t = 0$) is formally infinite. This arises from the unrealistic "sharp" corners in the momentum distribution which produces large wings in the configuration space probability density. For a more realistic momentum distribution, the wings would fall off faster and the mean square position would converge. Ignoring the large wings of the configuration space probability density, make a heuristic estimate of the position uncertainty. What is the momentum—position uncertainty product? Does this agree with the limit from the uncertainty principle?

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