1. The Deuteron. A deuteron (^{2}H nucleus) is a bound state of a neutron (charge 0, mass 939.5 MeV) and a proton (charge e, mass 938.2 MeV). Scattering measurements determine that the separation of the neutron and proton is about $a = 1.5\,\mathrm{fm}$ and the binding energy, determined from mass measurements, is $E_b = 2.226\,\mathrm{MeV}$. Approximate the potential energy as a spherical square well, $V(r) = -V_0$ for $r < a$ and $V(r) = 0$ for $r > a$.

(a) What is the value of V_0 in MeV?

(b) Can the deuteron have an excited (but still bound!) state with $l = 0$?

(c) What do you think about bound states with $l > 0$? (An elaborate calculation is not required!)

2. This problem appeared on the January, 2002 prelims. Unless I'm missing something, the condition asked for is a transcendental equation involving U_0.

A particle of mass m moves in the spherically symmetrical potential in 3 dimensions:

$$V(r) = \begin{cases} 0, & 0 \le r < a, \\ -U_0, & a < r < b, \\ 0, & b < r, \end{cases}$$

where $U_0 > 0$.

What is the condition on U_0 so that there will not be any bound states?

3. Hydrogen. For the H atom ground state wave function, compute the following expectation values.

$$\left\langle \frac{e^2}{r} \right\rangle , \quad \left\langle \frac{p^2}{2m} \right\rangle , \quad \langle r \rangle , \quad \langle p_r \rangle , \quad \langle x \rangle , \quad \langle p_x \rangle , \quad \Delta r \Delta p_r , \quad \Delta x \Delta p_x .$$

Recall, the wave function is

$$\psi(r, \theta, \phi) = \frac{1}{\sqrt{2}} \left(\frac{2}{a} \right)^{3/2} e^{-r/a} \frac{1}{\sqrt{4\pi}} ,$$

where $a = \hbar^2/me^2$ and m is the reduced mass of the electron and proton.

4. An electron outside liquid Helium. Based on a problem from Schwabl. The region $x < 0$ is filled with liquid helium and $x > 0$ is a vacuum. (Well, we have to ignore the vapor pressure of the helium!) An electron is at $x > 0$ and its potential energy is approximately described by $V(x) = +\infty$ for $x < 0$, due to the repulsion of the electron from the surface, and $V(x) = -Ze^2/x$ for $x > 0$ due to the image charge below the surface. $Z = (\epsilon - 1)/4(\epsilon + 1)$ where ϵ is the dielectric constant and for He, $\epsilon = 1.057$. The motion parallel to the surface is just that of a free particle, so we are only concerned with the motion perpendicular to the surface.

(a) Obtain expressions for the (bound state) energy eigenvalues and eigenfunctions.

(b) What is the numerical value of the ground state energy?

(c) Explicitly list the first four eigenfunctions. Be sure they are normalized.