1. The Deuteron. A deuteron ($^2$H nucleus) is a bound state of a neutron (charge 0, mass 939.5 MeV) and a proton (charge $e$, mass 938.2 MeV). Scattering measurements determine that the separation of the neutron and proton is about $a = 1.5$ fm and the binding energy, determined from mass measurements, is $E_b = 2.226$ MeV. Approximate the potential energy as a spherical square well, $V(r) = -V_0$ for $r < a$ and $V(r) = 0$ for $r > a$.

(a) What is the value of $V_0$ in MeV?

(b) Can the deuteron have an excited (but still bound!) state with $l = 0$?

(c) What do you think about bound states with $l > 0$? (An elaborate calculation is not required!)

2. This problem appeared on the January, 2002 prelims. Unless I’m missing something, the condition asked for is a transcendental equation involving $U_0$.

A particle of mass $m$ moves in the spherically symmetrical potential in 3 dimensions:

$$V(r) = \begin{cases} 
0, & 0 \leq r < a, \\
-U_0, & a < r < b, \\
0, & b < r,
\end{cases}$$

where $U_0 > 0$.

What is the condition on $U_0$ so that there will not be any bound states?

3. Hydrogen. For the H atom ground state wave function, compute the following expectation values.

$$\left\langle \frac{e^2}{r} \right\rangle, \quad \left\langle \frac{p^2}{2m} \right\rangle, \quad \langle r \rangle, \quad \langle p_r \rangle, \quad \langle x \rangle, \quad \langle p_x \rangle, \quad \Delta r \Delta p_r, \quad \Delta x \Delta p_x.$$  

Recall, the wave function is

$$\psi(r, \theta, \phi) = \frac{1}{\sqrt{2}} \left( \frac{2}{a} \right)^{3/2} e^{-r/a} \frac{1}{\sqrt{4\pi}},$$

where $a = h^2/2me^2$ and $m$ is the reduced mass of the electron and proton.
4. An electron outside liquid Helium. Based on a problem from Schwabl. The region 
$x < 0$ is filled with liquid helium and $x > 0$ is a vacuum. (Well, we have to ignore 
the vapor pressure of the helium!) An electron is at $x > 0$ and its potential energy is 
approximately described by $V(x) = +\infty$ for $x < 0$, due to the repulsion of the electron 
from the surface, and $V(x) = -Ze^2/x$ for $x > 0$ due to the image charge below the surface. 
$Z = (\epsilon - 1)/4(\epsilon + 1)$ where $\epsilon$ is the dielectric constant and for He, $\epsilon = 1.057$. The motion 
parallel to the surface is just that of a free particle, so we are only concerned with the 
motion perpendicular to the surface.

(a) Obtain expressions for the (bound state) energy eigenvalues and eigenfunctions.

(b) What is the numerical value of the ground state energy?

(c) Explicitly list the first four eigenfunctions. Be sure they are normalized.