PHYSICS DEPARTMENT, PRINCETON UNIVERSITY

PHYSICS 505 FINAL EXAMINATION

January 13, 2010, 1:30–4:30pm, Jadwin 343

This exam contains five problems. Work any three of the five problems. All problems count equally although some are harder than others. Do all the work you want graded in the separate exam books. *Indicate clearly which three problems you have worked and want graded.* I will only grade three problems. If you hand in more than three problems without indicating which three are to be graded, I will grade the first three, only!

The exam is closed everything: no books, no notes, no calculators, no computers, no cell phones, no ipods, etc.

Write legibly. If I can't read it, it doesn't count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*

If you finish early, do not leave your exam books in the room. Instead, take them to Ms. Angela Glenn in Room 231 Jadwin Hall.

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1. Two particles in a box. Two particles of mass m are confined to a rectangular box of sides $a < b < c$. They are in the lowest energy state compatible with the conditions in the cases below. For each of these cases, determine the lowest energy state and its energy and also use first order perturbation theory to determine the correction to the energy if there is an interaction between the particles of the form $V = (V_0(abc))\delta^{(3)}(\boldsymbol{r}_1 - \boldsymbol{r}_2)$.

(a) Two non-identical particles.

(b) Two identical particles of spin 0.

(c) Two identical particles of spin 1/2 in the singlet state.

(d) Two identical particles of spin 1/2 in the triplet state.

2. Coupled angular momentum states. A two particle system is in a state $|\psi_0\rangle$, where each particle has orbital angular momentum quantum numbers $l = 1$ and $m_l = 0$. The two particles are **not** identical. The total angular momentum is denoted by $\boldsymbol{J} = \boldsymbol{L}_1 + \boldsymbol{L}_2$ and the eigenvalues of J^2 are $\hbar^2\, j(j+1)$.

(a) The two particle state may be expanded in eigenstates of J^2. What values of j have non-zero amplitude in the expansion? For each of these values, what is the probability that it will be found in a measurement of J^2?

At $t = 0$, a coupling between the angular momenta is turned on so the system has Hamiltonian,

$$H = \gamma \boldsymbol{L}_1 \cdot \boldsymbol{L}_2 \,,$$

where γ is a constant. The state of the system now depends on time, $|\psi(t)\rangle$.

(b) Show that $|\,\langle\psi(t) \mid \psi_0\rangle\,|^2$ is periodic with period T and find T. Also evaluate $|\,\langle\psi(t) \mid \psi_0\rangle\,|^2$ at $t = T/2$

3. Electric pulse. Consider hydrogen in its ground state for $t \leq 0$. It's acted on by an electric field pulse in the z-direction of the form

$$\boldsymbol{E}(t) = \begin{cases} 0 & \text{if } t \leq 0 \\ E_0 \boldsymbol{e}_z e^{-t/\tau} & \text{if } t > 0 \end{cases} ,$$

where E_0 and τ are positive constants. Such a field can be produced by placing the atom between the plates of a capacitor and pulsing the capacitor. Obtain first order expressions for the probability that the atom winds up in a 2s state and the probability that the atom winds up in a 2p state at $t = +\infty$.

Useful data: for hydrogen,

$$\begin{aligned} R_{10}(r) &= 2\left(\frac{1}{a}\right)^{3/2} e^{-r/a} , \\ R_{20}(r) &= 2\left(\frac{1}{2a}\right)^{3/2} \left(1 - \frac{r}{2a}\right) e^{-r/2a} , \\ R_{21}(r) &= \frac{1}{\sqrt{3}}\left(\frac{1}{2a}\right)^{3/2} \frac{r}{a} e^{-r/2a} , \\ a &= \frac{\hbar^2}{me^2} . \end{aligned}$$

4. Particle in a box. A particle of mass m is confined to the cubical box $0 \le x \le L$, $0 \le y \le L$, $0 \le z \le L$ by a potential that is 0 inside the box and very large (∞) outside the box.

(a) What are the normalized wavefunctions and energies of the stationary states of the particle in this box? Be sure to give the quantum numbers necessary to specify the state.

(b) Consider an energy E such that $E \gg E_0$, where E_0 is the ground state energy. What is the density of states (number of states per unit energy) at energy $E + E_0 \approx E$?

(c) At $t = 0$, with the particle in its ground state, a weak, time dependent, potential is turned on:

$$V(\boldsymbol{x}, t) = V_0 L^3 \, \delta(x - L/2) \, \delta(y - L/2) \, \delta(z - L/2) \, \cos(\omega t) \, ,$$

where $\hbar\omega \gg E_0$. This potential is an oscillating "spike" at the center of the box. It can cause transitions from the ground state to excited states. What are the selection rules for these transitions? That is, what constraints must the quantum numbers of the excited states satisfy?

(d) What is the first order transition rate, from the ground state to the total of all excited states allowed by energy conservation and the selection rules, produced by the oscillating potential of part (c)?

5. Scattering from a spherical shell. A particle of mass m and energy $E = \hbar^2 k^2 / 2m$ is scattered by the fixed, spherically symmetric potential

$$V(r) = -V_0 a \delta(r - a) \, ,$$

where V_0 and a are a positive constants. In the following, use suitable approximations.

(a) What is the total scattering cross section at very low energies ($ka \ll 1$)?

(b) What is the differential cross section at very high energies ($ka \gg 1$)?

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