This exam is a take home exam. It must be taken in three continuous hours. That is, if you open the exam at time $t$, you must do no more work on the exam after time $t + 3$ hours. You must return your solution booklet to Laurel Lerner (room 210 Jadwin) the next day (regular business hours) after you pick up the exam from her. Exams may be picked up starting on January 16 and all exams must be completed by January 23, 2008.

The exam is closed books and notes. There are no numerical questions, so calculators, computers, cell phones, etc., are also not allowed.

This exam contains five problems. Work any three of the five problems. All problems count equally although some are harder than others. Do all the work you want graded in the separate exam booklets. *Indicate clearly which three problems you have worked and want graded.* I will only grade three problems. If you hand in more than three problems without indicating which three are to be graded, I will grade the first three, only!

Write legibly. If I can’t read it, it doesn’t count!

Put your name on all exam books that you hand in. (Only one should be necessary!!!) On the first exam book, rewrite and sign the honor pledge: *I pledge my honor that I have not violated the Honor Code during this examination.*
1. A particle in a 2D box. A particle is confined to a square box, $0 \leq x \leq L$ and $0 \leq y \leq L$. We are not interested in the $z$-motion, so this is a 2D problem.

(a) Obtain the energies and eigenfunctions. What is the degeneracy of the few lowest levels?

(b) A small perturbation $V = Cxy$, where $C$ is a constant, is applied. Find the energy change for the ground state and the first excited state in the lowest non-vanishing order. Construct the appropriate eigenfunctions in the case of the first excited state.

2. Zeeman splitting of valence electron states. Consider an atom with a single valence electron with orbital angular momentum quantum number $l$. Its total angular momentum quantum number is $j = l + 1/2$ or $j = l - 1/2$. The total angular momentum states are usually split by spin-orbit interactions producing $2j + 1$ degenerate states with the same $j$ but different $m$ (the quantum number for $J_z$). Application of a uniform magnetic field $B$ in the $z$ direction splits the magnetic substates. The Hamiltonian for the magnetic interaction is

$$H_1 = -\frac{eB}{2m_e c} (L_z + 2S_z) = -\frac{eB}{2m_e c} (J_z + S_z),$$

where $m_e$ is the mass of the electron, $J$, $L$, and $S$ are the total, orbital, and spin angular momentum operators, and the $g$-factor for the electron has been taken to be exactly 2. Then the Zeeman shift is

$$\delta E_{j,m} = -\frac{eB}{2m_e c} \langle jm | J_z + S_z | jm \rangle = -\frac{eB}{2m_e c} (m\hbar + \langle jm | S_z | jm \rangle).$$

To evaluate these shifts, we need to evaluate $\langle jm | S_z | jm \rangle$.

(a) Consider the special case $l = 1$. Construct the $j = 3/2$ and $j = 1/2$ states and evaluate $\langle jm | S_z | jm \rangle$ for all the states. Show that

$$\delta E_{j,m} = k_j m,$$

and state the two values of $k_j$ that you’ve just computed.

(b) Now consider the case for general $l$. Assume $\delta E_{j,m} = k_j^l m$ and determine $k_j^l$ for the two multiplets $j = l \pm 1/2$.

3. Crossed $E$ and $B$ fields. Consider a particle with mass $m$ and charge $e$ moving in uniform $B = Be_z$ and $E = Ee_x$ fields with $E < B$. Use the gauge $A = Bxe_y$. The Hamiltonian is

$$H = \frac{1}{2m} \left( p - \frac{e}{c} A \right)^2 - eEx.$$

Find the eigenfunctions and eigenvalues for this Hamiltonian. Note that if you should find that the eigenfunctions involve standard functions which we already know about, you
don’t have to write out each one explicitly. For example, if you should find (you won’t) that the eigenfunctions involve the Legendre polynomials $P_l(y)$, you can just leave $P_l(y)$ in your result rather than explicitly write out what $P_l(y)$ is. Of course, you have to say that $P_l$ stands for a Legendre polynomial of order $l$.

Hint: you might start by reminding yourself what the classical motion looks like.

4. Identical particles and a time dependent perturbation. Two identical, neutral, spin 1/2 particles, each of mass $2m$ and magnetic moment $\mu$ are constrained to move along a line. The interaction between them is spin dependent and described by the Hamiltonian

$$H = \frac{p_1^2}{4m} + \frac{p_2^2}{4m} + (2\hbar^2 - S_T^2)U(x_1 - x_2).$$

$S_T = S_1 + S_2$ is the total spin of the system whose quantum number $s_T$ may be 0 or 1. $U$ is an infinite well,

$$U(x) = \begin{cases} -\pi^2/8ma^2, & |x| < a, \\ \infty, & a < |x|. \end{cases}$$

(a) After separating off the center of mass motion, find the energy eigenstates for the relative motion of the system in zero magnetic field and the corresponding wave functions. What is the energy, $E_0$ of the ground state?

(b) Initially, the system is in its ground state. Electromagnetic radiation is applied to the system and the magnetic moments interact with the magnetic field of the wave which is

$$B_z = B \cos(kx - \omega t),$$

where $k = \omega/c$. Assume that $\hbar \omega + E_0 > 0$. What is the lifetime of the ground state to first order in perturbation theory? You may assume $ka \ll 1$ and expand to first order in $ka$.

5. Scattering from a spherical shell. A particle of mass $m$ and energy $E = \hbar^2k^2/2m$ is scattered by the fixed, spherically symmetric potential

$$V(r) = -V_0a\delta(r - a),$$

where $V_0$ and $a$ are a positive constants. In the following, use suitable approximations.

(a) What is the total scattering cross section at very low energies ($ka \ll 1$)?

(b) What is the differential cross section at very high energies ($ka \gg 1$)?
THIS PAGE INTENTIONALLY LEFT ALMOST BLANK